Engineered omnidirectional external-reflectivity spectra from one-dimensional layered interference filters

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Owing to optical refraction, external rays that are incident upon a high-refractive-index medium fall within a small internal cone angle. A one-dimensionally periodic Bragg structure can reflect over an angular acceptance range that is greater than the small internal refraction cone, if the internal refractive-index contrast is sufficient. Thus Winn *et al.* [Opt. Lett. (to be published)] charted the range of refractive indices at which omnidirectional external reflection occurs. A wide spectral gap requires a high-index contrast. It is proposed that, by chirping or grading the periodicity of the structure, one can cover an arbitrarily wide spectral range with only a modest index contrast and, furthermore, that arbitrary spectral shapes can be produced. The graded-periodicity approach requires only a modest index contrast, provided that the average refractive index is >2.

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One-dimensionally layered interference filters are notorious for having reflectivity spectra that are a function of angle. The Bragg condition $2a \cos\theta = m\lambda$ depends on $\cos\theta$, making it difficult to achieve high reflectivity for incidence angles θ covering the full range from 0 to 90°.

In a Bragg reflector of high average refractive index n_{av} the internal angles are restricted by Snell's law, $n_{\mathrm{av}} \sin\theta = 1$. Arbitrarily large external incidence angles fall within a cone of half-angle θ internally. Therefore one need only satisfy the Bragg condition within the angular range $\theta < \sin^{-1}(1/n_{\mathrm{av}})$ to ensure omnidirectional external reflectivity. Recently Winn *et al.*[1] determined the specific refractive-index requirements under which one-dimensionally layered interference filters can reflect over the full external angular range. For high n_{av}, the angle θ becomes small, and omnidirectional external reflectivity can be easily achieved.

Such omnidirectional external reflectivity should not be confused with the creation of a three-dimensional photonic bandgap.[2] A photonic bandgap requires omnidirectional reflection for both external and internal angles and demands full three-dimensional periodicity.

One-dimensionally periodic layered structures have traditionally been called interference filters. Their performance at oblique angles of incidence was analyzed in Ref. 3. Winn *et al.*[1] calculated the reflected optical forbidden gap for a given $n_{\mathrm{av}} = (n_1 + n_2)/2$ and a given index contrast $(n_1 - n_2)$, assuming that the external index is $n = 1$. The broader the desired frequency range, the greater the demands on index contrast $(n_1 - n_2)$.

There is another approach to solving the problem of achieving a broad reflection spectrum, which is to start with the least index contrast that produces the narrowest omnidirectional spectral reflector and then gradually to vary the interlayer spacing down through the successive layers, i.e., to chirp the layer period. Then, at some depth in the layer structure any optical frequency will find just that layer spacing that satisfies the Bragg reflection condition. Thus a huge spectral range could be covered, far more than would be allowed by even the largest index contrast that is available in nature. In addition, it will then be easy for one to adjust the reflectivity spectrum by dropping certain layer spacings from the one-dimensional stack. Then, those frequencies will be transmitted without satisfying the Bragg condition. An arbitrary spectral shape can thereby be engineered over a very large spectral bandwidth.

Thus we must find not the refractive indices that produce the widest omnidirectional reflectivity spectrum but the indices that produce the narrowest omnidirectional reflection spectrum. A good starting point is the normal-incidence forbidden[3] frequency gap:

$$\Delta\nu_{\mathrm{gap}}/\nu = (2/\pi)(n_1 - n_2)/n_{\mathrm{av}}\,, \tag{1}$$

where ν is the center frequency of the forbidden gap. Equation (1) can be reinterpreted in terms of a forbidden angular range if the photonic valence band edge is at normal incidence and then shifts to higher frequency when off normal, finally matching the normal-incidence conduction band, then the maximum range of forbidden internal angles is

$$\cos\theta \approx 1 - (\Delta\nu_{\mathrm{gap}}/\nu)\,. \tag{2}$$

By substituting Eq. (1) into relation (2) and performing a Taylor expansion on the cosine, one can approximate the forbidden internal angle as

$$\theta \approx \left(\frac{4}{\pi}\,\frac{n_1 - n_2}{n_{\mathrm{av}}}\right)^{1/2}. \tag{3}$$

When this angle becomes greater than the angle of refraction, $\theta = \sin^{-1}(1/n_{\mathrm{av}}) \approx (1/n_{\mathrm{av}})$, the external reflectivity becomes omnidirectional. This condition is fulfilled when

$$(n_1 - n_2) \geq \frac{\pi}{4n_{\mathrm{av}}}\,. \tag{4}$$

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Relation (4) is the minimal required index contrast for external omnidirectionality at a single optical frequency. If the narrowest spectral features are desired, relation (4) need be only minimally satisfied. For an average refractive index $n_{\mathrm{av}} = 2$ the required index contrast is only $(n_1 - n_2) = 0.39$. At $n_{\mathrm{av}} = 3$ the required index contrast is only $(n_1 - n_2) = 0.26$, which is easily satisfied, for example, by the AlGaAs/GaAs mirror structures that are incorporated into vertical-cavity surface-emitting lasers.[4] Thus the mirrors in these lasers are usually externally omnidirectional!

The frequency edges of the forbidden band are sharp, but they shift by cos θ as a function of angle. The opposite band edges would coincide in frequency when the relative change in cos θ is the same as the relative forbidden gap:

$$\Delta\nu_{\mathrm{gap}}/\nu \approx (1 - \cos\theta) \approx \frac{1}{2{n_{\mathrm{av}}}^2}\,. \tag{5}$$

In this concept the frequency coverage is made complete by means of chirping or grading the layer periodicity. The angular variation defines how sharply the edges of the reflection spectrum and other spectral features can be defined. That spectral sharpness is defined by Eq. (5) and can produce a spectral resolution as sharp as 6%, versus external angle, for $n_{\mathrm{av}} = 3$. Clearly there are benefits in increasing the average refractive index while keeping the index contrast down to its minimum permissible value, relation (4).

In this treatment polarization effects were neglected. It is known that p-polarized light at oblique incidence can have a forbidden gap that is less than the normal incidence gap given in Eq. (1). At the Brewster angle tan $\theta = n_1/n_2$ the gap disappears entirely. However, the Brewster angle is always greater than 45°. For an average index of >2 the refraction angle is always $\ll 45°$. Therefore, under the conditions of a substantial average refractive index, the polarization effects produce only a small correction to the formulas presented above.

In grading the layer spacing, how many layers are required for achievement of a certain reflectivity in each spectral band? The reflectivity of a quarter-wave multilayer stacked structure can be approximated[5] as

$$\mathrm{Reflectivity} = 1 - \exp\!\left(-2\,\frac{n_1 - n_2}{n_{\mathrm{av}}}\,N\right), \tag{6}$$

where N is the number of layer periods. For a reflectivity of 90% the exponent has to equal 2.303. Thus the requirement can be satisfied when $N = 2.303 \times n_{\mathrm{av}}/2(n_1 - n_2)$. Inserting the index contrast from Eq. (4), for marginal omnidirectionality but maximal spectral resolution, we can write this as $N = 2.303 \times 2(n_{\mathrm{av}})^2/\pi$, which would translate to 12 periods to produce a frequency notch with the sharpest possible spectral resolution, $\approx$6% at $n_{\mathrm{av}} = 3$. The layer spacing could then be graded or chirped to the next desired frequency band. If a particular periodic spacing is absent, then that range of frequencies will not be Bragg reflected. In this way an engineered reflectivity spectrum can be created.[6]

The conditions for producing an omnidirectional reflectivity spectrum from a layered interference filter have been analyzed. It was found that with the concept described above the spectrum will shift by only 6% over the full range of external angles and that angle tuning will determine the angle-averaged spectral resolution of the narrowest spectral features. Emphasis was placed on a moderately high average refractive index, in combination with the smallest permissible index contrast, as given by relation (4).

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