High-Q Radio-Frequency Structures Using One-Dimensionally Periodic Metallic Films

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***Abstract*— High-Q structures are very interesting theoretically, and very important practically, for a variety of engineering applications in communication systems. We address the issue of designing a thin-film metal structure of reflectivity higher than the intrinsic reflectivity of the bulk metal itself. We study a finite array of planar conducting layers of arbitrary thickness periodically placed an arbitrary distance apart, and we arrive at an exact analytical formula for the reflection and transmission coefficients. These structures are equivalent to a one-dimensional metallic photonic bandgap (PBG) system. We apply our formulas to the microwave regime and fully explore the system's three-dimensional parameter space, consisting of the number of layers, their thickness, and their spacing. We find very significant enhancements of the radio frequency-Q, relative to the bulk metal, in narrow regions of the parameter space.**

***Index Terms*— High-Q structures, periodic metallic films, photonic bandgap.**

I. Introduction

Metallic conductors are excellent reflectors at microwave frequencies. Their reflectivity is determined by their conductivity, which is a physical characteristic of the material. An important issue is whether it is possible to design a metallic periodic structure of reflectivity higher than the intrinsic reflectivity of the bulk metal itself. It is well known that if a dielectric material of modest intrinsic reflectivity is considered, one can design a multilayer structure of dielectric films whose reflectivity is far greater than the intrinsic reflectivity of the bulk dielectric [1], [2]. The purpose of this paper is to examine whether this is true for metals, whose reflectivity is very high to start with, even for a single thin film [3], as well as determining the regions of parameter space where this is possible.

The fundamental thin-film-thickness scale length for making a good metallic reflector is not actually the "skin depth," but rather the smaller thickness which corresponds to 377-Ω^2. That thickness is more than a 1000 times less than a skin depth, and can be of the order of angstroms.

By inserting such ultrathin layers at the nodes of a standing-wave electromagnetic-mode pattern, the electric field within the metal film can be made tiny, leading to small dissipation of energy and high-Q. In fact, the energy dissipation per metallic layer at a node scales with the thickness of that layer cubed. At the same time, the reflectivity of each layer is proportional to the thickness per layer since it depends on the phase shift between front and back surfaces. Therefore, by making the layers thin enough, and using enough layers, an arbitrarily high reflectivity can be achieved.

Thus, employing geometry to enhance the properties of normal metals, such ultrahigh-reflectivity metallic structures could be fashioned into high-Q cavities with much higher performance than ordinary metallic cavities.

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II. N-Layer System

Our general system consists of an arbitrary number (N) of identical conducting layers of conducting material, of thickness d, spaced within a material d' distance apart. Material can be arbitrary, but we will consider air gaps for this paper. There are a total of $2N+1$ regions with $2N$ interfaces, and we focus on normal incidence.

A. Matrix Solution of the Problem

The boundary conditions on all interfaces can be solved according to a transfer-matrix formulation [4]. The following transfer matrix F, connecting two adjacent unit cells, can be found:

$$\begin{aligned} F &= u \times U \\ u &= \left(\frac{1-\Gamma_{\rm in}^2 e^{-2k_b d}}{1-\Gamma_{\rm in}^2}\right) e^{k_a d'} e^{k_b d} \\ U &= \begin{pmatrix} 1 & -e^{-2k_a d'}\Gamma_1 \\ \Gamma_1 & z \end{pmatrix} \end{aligned} \tag{1}$$

where

$$\begin{aligned} \Gamma_1 &= \frac{\Gamma_{\rm in}(1-e^{-2k_b d})}{1-\Gamma_{\rm in}^2 e^{-2k_b d}} \\ z &\equiv e^{-2k_a d'}\left(\frac{e^{-2k_b d}-\Gamma_{\rm in}^2}{1-e^{-2k_b d}\Gamma_{\rm in}^2}\right) \\ \Gamma_{\rm in} &\simeq \frac{\dfrac{1+j}{2\Delta}-1}{\dfrac{1+j}{2\Delta}+1} \\ \Delta &\equiv \sqrt{\frac{\sigma}{2\epsilon_0\omega}}. \end{aligned} \tag{2}$$

In (2), Γ_1, $\Gamma_{\rm in}$ are the reflection coefficients of a single metal film and intrinsic (bulk) metal, respectively, σ is the conductivity of the metal, and the skin depth $\delta = \lambda_0/(2\pi\Delta)$. We will parameterize the propagation constants by

$$k_a d' = j\frac{2\pi d'}{\lambda_0} \equiv jy \quad k_b d = (1+j)\frac{d}{\delta} \equiv (1+j)x. \tag{3}$$

Therefore, the reflection and transmission coefficients are

$$\Gamma_N = \frac{(U^N)_{21}}{(U^N)_{11}} \quad T_N = u^{-N} e^{k_a d'} \frac{1}{(U^N)_{11}}. \tag{4}$$

The Nth power of the matrix U can be calculated analytically. The result is shown in (5) and (6), at the bottom of the following page.

Similarly, we can write the transmission coefficient as

$$\begin{aligned} T_N = \Gamma_N \times \frac{1-\Gamma_{\rm in}^2}{\Gamma_{\rm in}} \frac{1}{\sinh(k_b d)} \frac{\zeta}{1+\zeta} \\ \cdot \left[1-\left(\frac{1-\zeta}{1+\zeta}\right)^N\right]^{-1} \left[\frac{u(1+z)(1+\zeta)}{2}\right]^{1-N}. \end{aligned} \tag{7}$$

The $N=\infty$ limit may also be readily calculated from these formulas. In this limit, the system is equivalent to an artificial one-dimensional photonic crystal, which is hopefully exhibiting photonic bandgap (PBG) behavior [5]. This limit is important for analytically isolating the resonant regions of the parameter space, understanding better the resonant behavior of the system as the number of layers increases, and deriving simplified closed-form approximate expressions for the design parameters in the resonant regions. We find

$$\Gamma_\infty = \Gamma_1 \times \frac{2}{\zeta(1+z)+(1-z)}, \qquad T_\infty = 0. \tag{8}$$

We will focus on the unloaded Q-factor (finesse) of the system $Q = 1/(1-|\Gamma|^2)$.

B. The Resonant Regions

We expect the resonant regions to be at the vicinity of $y \simeq \pi \to d' \simeq \lambda_0/2$. Since the layers are conducting and the metal has very high intrinsic reflectivity, we expect narrow-band resonances. Let us parameterize the resonant regions in y by

$$y = \pi - \frac{\epsilon}{2} \quad \frac{1}{\Delta} = \tau\frac{\epsilon}{2} \tag{9}$$

where $\epsilon \ll 1, \tau$ is a multiplicative parameter, which can be $\mathcal{O}(1)$ or higher, and we have parameterized the other small parameter $1/\Delta$ of the problem in terms of ϵ. Taylor-expanding Γ_{in}, Γ_1 up to $\mathcal{O}(\epsilon^2)$, we get (10), shown at the bottom of this page, for the Q-factor ratio, valid in the range $x \geq 0.1$, where

$$R_\pm(x) \equiv \frac{1-e^{-4x} \pm 2e^{-2x}\sin 2x}{1+e^{-4x}-2e^{-2x}\cos 2x}. \tag{11}$$

III. Numerical Results and Conclusions

In this section, we present numerical results for the Q-factor ratio Q_s/Q_{in}, where the system (s) will be either an N-layer or ∞-layer system. In Fig. 1, we probe the thicknesses $x \in \{0.01, 1\}$, with corresponding resonances at $y \in \{\pi - 1/(90\Delta), \pi - 1/\Delta\}$, and Q enhanced by as much as a factor of 170. The peak values are fitted very well by the equation

$$\frac{Q_\infty^{\max}}{Q_{\text{in}}} = \sqrt{\pi}\frac{\delta}{d}. \tag{12}$$

Regarding the (x, y) parameter space, the relation

$$y = \pi - \frac{x^{1-\epsilon}}{\Delta} \tag{13}$$

with $\epsilon = 0.0035$ fits the envelope very well for $x \geq 0.1$. For $\epsilon = 0$, which corresponds to $d + d' = \lambda_0/2$, (13) fits the envelope well for $x \geq 0.2$. For smaller thicknesses, our general formulas show that the resonances are in the region

$$\frac{d'}{\lambda_0} \simeq \frac{1}{2} - \frac{1}{\tau}\,\frac{\delta}{\lambda_0} \tag{14}$$

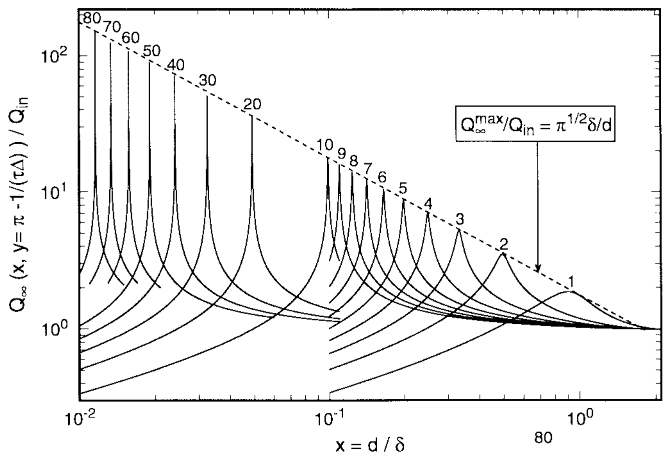


Fig. 1. The Q-factor ratio for the ∞-layer system in the thickness region $x \in \{0.01, 2\}$ and for resonances $y = \pi - 1/(\tau\Delta)$ (numbered curves correspond to different values of τ). The fitted envelope $Q_\infty^{\max}/Q_{\text{in}} \simeq \sqrt{\pi}\delta/d$ is also shown.

where τ is a discretized parametrization of the resonances for given thickness values

$$\begin{aligned} 0.1 \leq \frac{d}{\delta} \leq 1,\ 10 > \tau \geq 1 \\ 10^{-2} \leq \frac{d}{\delta} < 0.1,\ 90 > \tau \geq 10 \\ 10^{-3} < \frac{d}{\delta} < 10^{-2},\ 500 > \tau \geq 90. \end{aligned} \tag{15}$$

In Fig. 2, we show the power balance of the system of N layers in the thickness region $x \in \{0.1, 2\}$ when the distance of two successive layers is continuously optimized to ensure resonant behavior, as shown in (13).

The final design issue is the number of layers N necessary for a given gain (see Fig. 3). For a fixed number of layers N, the maximum possible Q-enhancement is given by

$$\frac{Q_N^{\max}}{Q_{\text{in}}} = \sqrt{N} \tag{16}$$

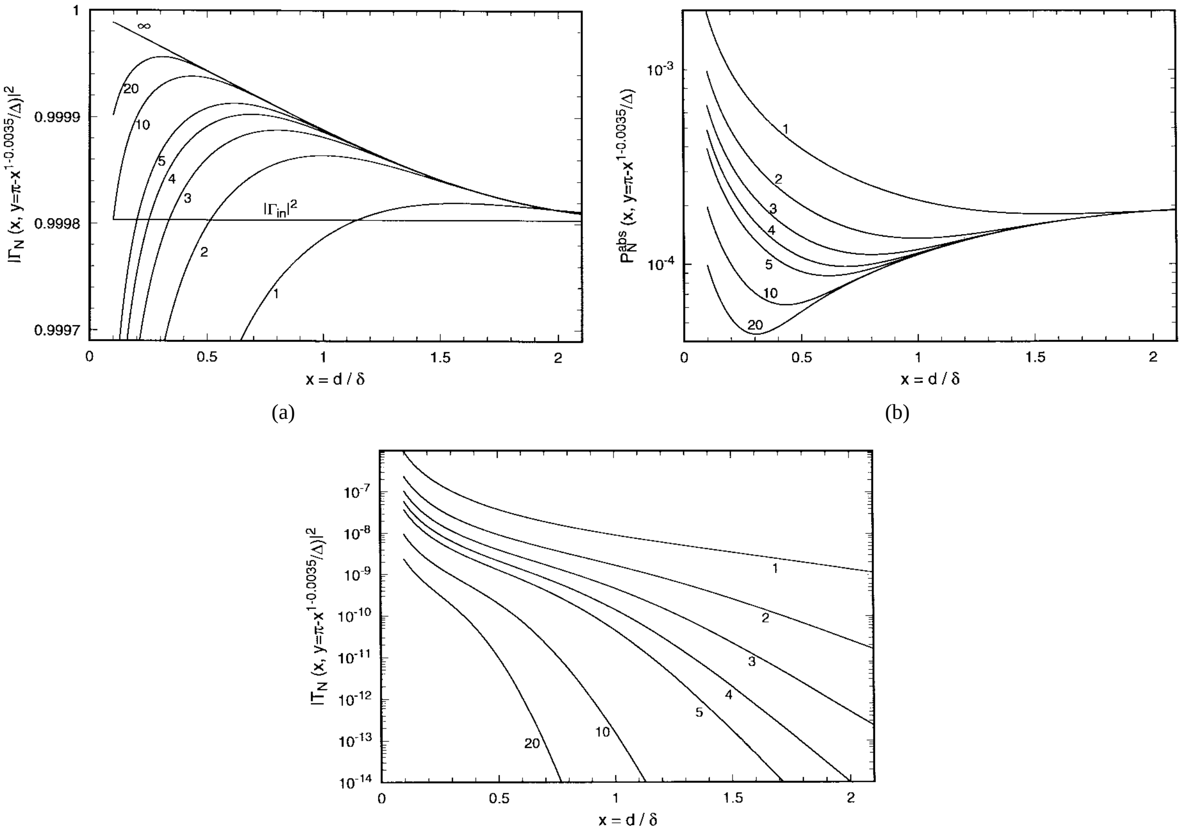
where, for large enough N, $(N \geq 5)$ layer thickness and spacing are given by the design formulas for the $N = \infty$ case above.

In conclusion, we have shown that by using spatial periodicity it is possible to make metal resonators with much higher Q-factors than those of cavities constructed by plain metal walls. We believe that our findings will have important applications on systems where it is necessary to exceed the customary Q of normal metallic structures.

$$\Gamma_N = \Gamma_1 \times \frac{2\Big[(1+\zeta)^N - (1-\zeta)^N\Big]}{(1-z)\Big[(1+\zeta)^N - (1-\zeta)^N\Big] + \zeta(1+z)\Big[(1+\zeta)^N + (1-\zeta)^N\Big]} \tag{5}$$

$$\zeta = \left(\frac{1-z}{1+z}\right)\sqrt{1 - e^{-2k_a d'}\left(\frac{2\Gamma_1}{1-z}\right)^2} \tag{6}$$

$$\frac{Q_\infty}{Q_{\text{in}}} \simeq \frac{1}{\sqrt{\dfrac{R_-(x)}{\tau} - \dfrac{1}{2\tau^2} + \sqrt{\left(\dfrac{R_-(x)}{\tau} - \dfrac{1}{2\tau^2}\right)^2 + \left(\dfrac{R_+(x)}{\tau} - 1\right)^2}}} \tag{10}$$



(c)

Fig. 2. (a) Reflected power for the N-layer system for various N (numbered curves), for thicknesses $x \geq 0.1$, and distances optimized continuously at the resonance through $y = \pi - x^{1-0.0035}/\Delta$. These results are for Cu ($\sigma = 5.76$ S/m) at $f = 5$ GHz. The intrinsic reflectivity is also shown. (b) Same as in (a) for absorption. (c) Same as in (a) for transmission.

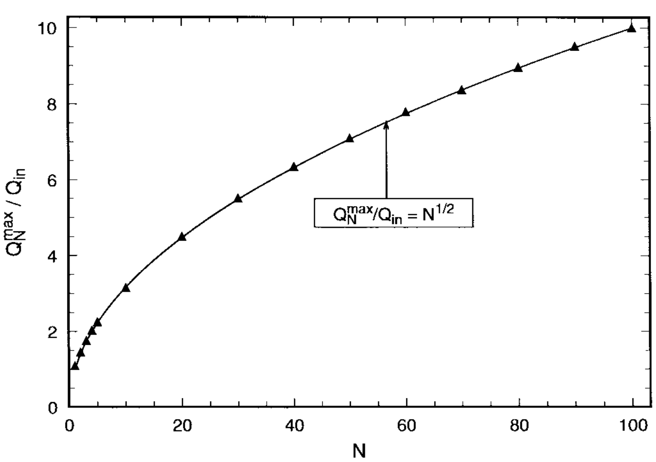


Fig. 3. Maximum Q-factor enhancement as a function of number of layers N. Each discrete data point is individually maximized in layer thickness (d) and spacing (d') for the corresponding fixed value of N. The solid line $Q_N^{\max}/Q_{\text{in}} = \sqrt{N}$ fits the data very well, except for the first $N = 1$ point, which overshoots the fit by 8%. Strictly speaking, that point is not a multilayer system.

One example is macroscopic cavities used for material measurements at millimeter waves. If one cavity dimension is $\simeq$10 cm, or $\simeq$130 half-wavelengths at 200 GHz, the loaded Q-factor would only increase by 10% for an increase of that linear dimension by an extra 14 half-wavelengths. On the other hand, with our design, the same volume increase would accomplish a loaded-Q enhancement of $\simeq \sqrt{N = 7} = 260\%$ (seven-layer shorts on each side).

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