## Analysis of Frequency Selective Layers via a Combined Finite-Element Integral-Equation Method (FE-IEM)

Lijun Zhang<sup>a</sup>, Harry Contopanagos<sup>a</sup>, Nicolaos G. Alexopoulos<sup>b</sup> and Eli Yablonovitch<sup>a</sup>

<sup>a</sup>Department of Electrical Engineering, UCLA

Los Angeles, CA 90095

<sup>b</sup>Department of Electrical and Computer Engineering, UCI

Irvine, CA 92697

#### Abstract

Frequency Selective Surfaces (FSS) made up by stacking Frequency Selective Layers (FSL) are analyzed via a Combined Finite-Element Integral-Equation Method (FE-IEM). These structures are more flexible in frequency selectivity than conventional FSS. Depending on the unit cell structure of the FSL, adjustable transmission gaps are found.

### 1. Introduction

Photonic Band Gap (PBG) structures have received great attention recently due to their unique frequency selective properties. If the operating frequency falls into the bandgap region, such kind of material may be ideal candidates for printed antenna applications because of suppression of surface wave modes at least in certain directions. Further, the bandgap information of the structure is closely related to its frequency selective properties, and because the bandgap characteristics depend on a large number of parameters determining the underlying artificial crystal, these structures are much more flexible in frequency selectivity than conventional Frequency Selective Surface (FSS).

For the FSS, the Method of Moments (MoM) combined with the surface integral equation has been a very powerful technique [1]. The PBG structures analyzed here are made up of several Frequency Selective Layers (FSL), and because there exists complexity in one unit cell, numerical methods which can treat material complexity need to be used, such as volume integral equation [2], finite element method [3,4] or FDTD [5]. In [6], the combined FE-MoM is used to treat two dimensional periodic structures. In this paper, we present our own hybrid FE-IEM for analyzing three-dimensional periodic problems, with emphasis in novel FSS made up of FSL. Our analysis and code are an evolution of Antilla's etal fromulation [7]. The developed code has the capability of synthesizing FSS by stacking FSLs rotated with respect to each other, resulting in extra frequency selectivity flexibility. In a companion paper, we present a sample of results for printed antenna application [8].

# 2. Basic Formulation

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The typical structure of a FSL is shown in Fig. 1. The basic idea of the analysis is to apply FEM to the inner region (one unit cell) while the boundary condition is given accurately by the integral equation, and all the field solution should satisfy Floquet's boundary condition.

### 2.1 FEM Formulation

The EM fields in the unit cell must satisfy Maxwell's equations and Floquet's boundary condition as well. The equivalent functional can be expressed as

$$F(\vec{E}) = \int \int \int_{V} [(\nabla \times \vec{E}) \cdot (\nabla \times \vec{E}^{\star}) - k_{0}^{2} \epsilon_{r}^{\star} \vec{E} \cdot \vec{E}^{\star}] \ dv - j k_{0} Z_{0} \int \int_{s} \vec{E} \cdot (\hat{n} \times \vec{H}^{\star}) ds. \tag{1}$$

## 2.2 IEM Formulation

The magnetic field integral equation can be written in the following

$$\int \int_{s} \vec{W}(\vec{R}) \cdot [\hat{n} \times \vec{H}^{i}(\vec{R})] ds = \int \int_{s} \vec{W}(\vec{R}) \cdot [\hat{n} \times \vec{H}(\vec{R})]/2 ds 
+ jk_{0}Y_{0} \int \int_{s} \vec{W}(\vec{R}) \cdot \hat{n} \times \int \int_{s'} g_{p}(\vec{R}, \vec{R}') \vec{M}(\vec{R}') ds' ds 
- 1/(jk_{0}Z_{0}) \int \int_{s} \vec{W}(\vec{R}) \cdot \hat{n} \times \nabla\nabla \cdot \int \int_{s'} g_{p}(\vec{R}, \vec{R}') \vec{M}(\vec{R}') ds' ds. \quad (2)$$

Here,  $\vec{M}(\vec{R}')$  is the magnetic current and  $g_p$  is the spatial periodic Green's function

$$g_{p} = \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} e^{-jk_{0}R_{p}}/(4\pi R_{p})e^{-jk_{0x}ma-jk_{0y}nb}.$$
 (3)

By applying Poisson's summation formula twice, the last two terms in equation (2) can be written into the following form

$$jk_0Y_0 \int \int_{s} \vec{W}(\vec{R}) \cdot \hat{n} \times \int \int_{s'} \widetilde{g_p}(\vec{R}, \vec{R}') \cdot \vec{M}(\vec{R}') ds' ds$$
$$-1/(jk_0Z_0) \int \int_{s} \vec{W}(\vec{R}) \cdot \hat{n} \times \nabla \nabla \cdot \int \int_{s'} \widetilde{g_p}(\vec{R}, \vec{R}') \vec{M}(\vec{R}') ds' ds, \quad (4)$$

where

$$\widetilde{g_p}(\vec{R}, \vec{R}') = 1/(2ab) \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} e^{-j\beta_{xm}(x-x') - j\beta_{yn}(y-y') - j\beta_{xmn}(z-z')} / (j\beta_{zmn}) \quad (5)$$

and  $\beta_{zm}$ ,  $\beta_{yn}$  and  $\beta_{zmn}$  are the propagation constants of the corresponding  $mn^{th}$  space harmonic in three directions respectively.

# 3. Numerical Results and Disscussion

The first numerical example is a dielectric slab with material blocks implanted periodically. Two kinds of material blocks are considered, one is a high-contrast dielectric while the other is a PEC, see Fig. 2. It can been seen that use of metallic blocks can stabilize the transmission band with respect to incident angle, and therefore this system is a much better FSS than its dielectric counterpart.

The second numerical case shown in Fig. 3 originated from the metallo-dielectric Photonic Band Gap structure presented in [5]. Such a structure can have a large bandgap by intentionally incorporating very strong capacitive interactions between metallic plates. Since in this paper we are interested in thin structures, only a small number of FSL's are considered. It is found that the two-layer case has a larger transmission band than its one-layer counterpart.

Our current work on FSS is focusing on the frequency selective characteristics of PBG strutures made up of multilayer FSL which are rotated with respect to each other (variable capacitive interactions) and connected with via holes.

#### 4. References

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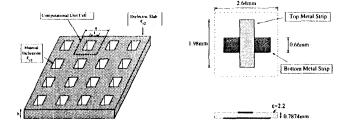


Figure 1: Typical Frequency Selective Layer. Left: Material blocks periodically implanted into a dielectric slab; Right: One unit cell of crossed metal strips sandwiched by a dielectric layer (top and side views are shown).

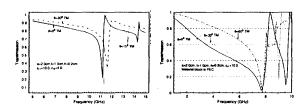


Figure 2: FSL: Material Blocks in a Dielectric Slab

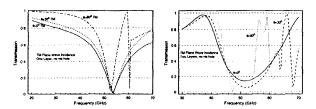


Figure 3: FSL composed of crossed Metal Strips Sandwiched by a Dielectric Slab. Left: One layer; Right: Two layers separated by a thin dielectric spacer  $(0.0254 \mathrm{mm})$