

# Optical Multiplexing using Transparency Window of Good Conductors.

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## Abstract

Thin metal films are excellent reflectors and extremely opaque at microwave and mm-wave frequencies. Bulk metal however becomes partly transparent at optical frequencies due to the shape of the Lorentz-Drude permittivity function at that frequency regime. We explore this property for designing optical filters composed of a finite number of periodically placed thin metal films supported by conventional dielectrics. The resulting filters exhibit comb-filter multiplexing characteristics similar to Bragg reflectors at frequencies higher than  $10^{14}$  Hz but unlike Bragg reflectors are completely opaque to any smaller frequencies.

## Introduction

It is well known that good conductors are excellent reflectors and extremely poor transmitters of electromagnetic radiation for frequencies up to mm-waves. It is perhaps less well-known that good conductors become transparent in a region around the free-electron plasma frequency  $\omega_p$  given by  $\omega_p^2 = Ne^2/m\epsilon_0$  where  $N$  is the free-electron density in the metal and  $m$  the electron effective mass. The plasma frequency falls in general in the optical regime for all good conductors and the corresponding relative complex permittivity of the metal can be obtained from the Lorentz harmonic oscillator model by setting the oscillator restoring force to zero, i.e.  $\omega_0 = 0$ :

$$\epsilon_c = 1 - \frac{\omega_p^2}{\omega(\omega - j\gamma)}. \quad (1)$$

Eq. (1) is valid all the way down to DC. The permittivity parameters for all good conductors are in the ranges  $7eV < \hbar\omega_p/2\pi < 15 eV$  and  $0.05 eV < \hbar\gamma/2\pi < 0.13 eV$  and result from fits to somewhat model-dependent low-energy reflectivity data [1]. For the rest of this paper we will choose the representative values

$$\omega_p = 10^{16} \text{ Hz}, \quad \gamma = 10^{-2}\omega_p, \quad (2)$$

giving a DC conductivity  $\sigma = \omega_p^2\epsilon_0/\gamma \approx \mathcal{O}(10^7 \text{ S/m})$ .

It has been recently observed [2] that one could use the transparency window of normal metals to construct metallo-dielectric periodic arrays with tunable transmission characteristics where the transmission through the periodic structure is enhanced relative to its single-film value much like in a Fabry-Perot resonator. The authors in [2] present various predictions of the transmittivity of these structures for normal plane-wave incidence a fixed number of unit cells and fixed values of dielectric thickness in the unit cell but do not provide a systematic investigation of the parameter-space for the filter performance in terms of its scaling variables.

In this paper we follow-up on the work of ref. [2] in two important ways: First we provide a closed analytical formula for the transmission and the reflection through an arbitrary number  $N$  of unit cells comprising the filter at an arbitrary angle of incidence and for both TE and TM polarizations. Secondly we systematically evaluate the filter performance and multiplexing properties as a function of the physical parameters of the filter namely material thicknesses polarization and incident angle and  $N$ .

### The Filter Design equations

We will denote with  $\{d_c, d_d\}$  the thicknesses of the conducting film and dielectric slab in the filter's unit cell and our filter will contain an arbitrary number  $N$  of such unit cells.

We find the reflection coefficient for  $p$ -polarized plane-wave incidence at angle  $\theta$  ( $p = 1 \rightarrow \text{TE}$ ,  $p = 2 \rightarrow \text{TM}$ ) to be [3]

$$\Gamma_N^p(\theta) = \frac{2f_{21} [(1 + \zeta)^N - (1 - \zeta)^N]}{(f_{11} - f_{22}) [(1 + \zeta)^N - (1 - \zeta)^N] + \zeta(f_{11} + f_{22}) [(1 + \zeta)^N + (1 - \zeta)^N]}, \quad (3)$$

where

$$\zeta = \frac{f_{11} - f_{22}}{f_{11} + f_{22}} \sqrt{1 + \frac{4f_{12}f_{21}}{(f_{11} - f_{22})^2}}. \quad (4)$$

The matrix elements  $f_{ij}$  are obtained from the transmission matrix through one unit cell of the filter:

$$\mathbf{f} = \begin{pmatrix} 1 - \Gamma_{a,c}^{1;p} \Gamma_{a,d}^{1;p} & -(\Gamma_{a,d}^{1;p} + \Gamma_{a,c}^{1;p} Z_{a,d}^p) \\ \Gamma_{a,c}^{1;p} + \Gamma_{a,d}^{1;p} Z_{a,c}^p & -\Gamma_{a,c}^{1;p} \Gamma_{a,d}^{1;p} + Z_{a,c}^p Z_{a,d}^p \end{pmatrix}, \quad (5)$$

where the functions  $\Gamma_{a,j}^{1;p}$  are the reflection coefficients of 1 slab of material  $j \in \{\text{conductor} \equiv c, \text{dielectric} \equiv d\}$  for  $p$ -polarized plane-wave oblique incidence from air

$$\Gamma_{a,j}^{1;p} = \Gamma_{a,j}^p \left( \frac{1 - e^{-2\gamma_j d_j}}{1 - (\Gamma_{a,j}^p)^2 e^{-2\gamma_j d_j}} \right), \quad (6)$$

and  $Z_{a,j}^p$  are closely related (but irreducible) functions

$$Z_{a,j}^p = \frac{-(\Gamma_{a,j}^p)^2 + e^{-2\gamma_j d_j}}{1 - (\Gamma_{a,j}^p)^2 e^{-2\gamma_j d_j}}. \quad (7)$$

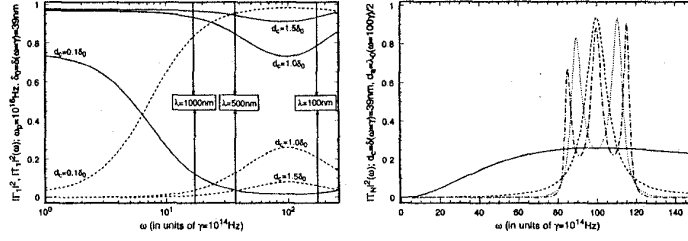


Figure 1: Left: Reflectivity (solid) and Transmittivity (dashed) of a single metal film  $\Gamma$  versus  $\omega\Gamma$  for various film thicknesses parametrized in terms of conductor skin depth  $\delta_0$  (evaluated at  $\omega = \gamma$ ); Right: Transmittivity for fixed unit cell thicknesses  $\Gamma$  as a function of  $N$  and  $\omega$ .  $N=1$  (solid)  $\Gamma$  2 (dashed)  $\Gamma$  3 (dotted)  $\Gamma$  4 (dot-dashed).

In these equations appear the intrinsic (bulk) reflection coefficients of the materials  $\Gamma_{a,j}^p$  and propagation constants  $\Gamma_j$  for the above excitation:

$$\Gamma_{a,j}^{p=1} = \frac{(\eta_j / \cos \theta_j) - (1 / \cos \theta)}{(\eta_j / \cos \theta_j) + (1 / \cos \theta)}, \quad \Gamma_{a,j}^{p=2} = \frac{\eta_j \cos \theta_j - \cos \theta}{\eta_j \cos \theta_j + \cos \theta}, \quad \gamma_j = j k_0 \cos \theta_j / \eta_j, \quad (8)$$

where  $\eta_j$  are the relative wave impedances

$$\eta_c = \frac{1}{\sqrt{\epsilon_c}}, \quad \eta_d = \frac{1}{\sqrt{\epsilon_d \sqrt{1 - j\alpha}}}, \quad (9)$$

and  $\epsilon_c, \epsilon_d, \alpha$  are the conductor permittivity given by Eq. (1) the real part of the permittivity and the loss tangent of the dielectric respectively and

$$\cos \theta_j = \sqrt{1 - \eta_j^2 \sin^2 \theta}. \quad (10)$$

The transmission coefficient through the filter  $\Gamma_N^p(\theta)$  is similarly calculated:

$$T_N^p(\theta) = \Gamma_N^p(\theta) \times \left( \frac{f_{11} + f_{22}}{f_{21}} \right) \zeta \left( \frac{2\Gamma_{a,c}^{1,p}\Gamma_{a,d}^{1,p}e^{-\gamma_c d_c}e^{-\gamma_d d_d}}{\Gamma_{a,c}^p\Gamma_{a,d}^p(f_{11} + f_{22})(1 + \zeta)} \right)^N \left[ 1 - \left( \frac{1 - \zeta}{1 + \zeta} \right)^N \right]^{-1}. \quad (11)$$

### A sample of results

Due to space limitations we will only present results for normal incidence and air-suspended metallic films.

In Fig. 1 we show the energy reflection and transmission of a single metal film for various film thicknesses. We observe that a single film becomes completely opaque at frequencies  $\omega < \gamma$  Even when the thickness is much smaller than the

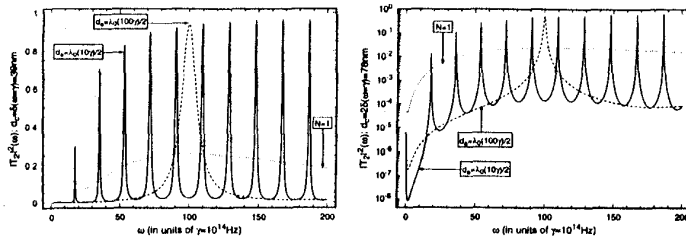


Figure 2: Multiplexing at optical frequencies  $\Gamma$  for different air (dielectric) thicknesses  $d_a$ . Left: For film thickness  $\delta_0$ ; Right: For film thickness  $2\delta_0$ .

skin depth at these smaller frequencies. For microwave frequencies  $\Gamma$  the skin depth of good conductors is of the order of microns  $\Gamma$  a full 2-3 orders of magnitude larger than the thicknesses shown here. We also show the basic filter response as a function of  $N$ . We observe that 2 unit cells provide the cleanest filter response.

In Fig. 2 we show the optical multiplexing properties of the filter. Notice that the periodicity of the response is controlled by the air (dielectric) thickness  $\Gamma$  which we normalize as a half-wavelength at an arbitrary frequency  $(\lambda_0(\omega)/2)$ . The response is that of a comb filter with peaks modulated according to the transmittivity of the single metal film. In the passbands  $\Gamma$  however  $\Gamma$  the transmittivity greatly overshoots the values of its single-film counterpart. In particular  $\Gamma$  for  $d_a = 2\delta_0$   $\Gamma$  the single-film transmittivity is less than 2%  $\Gamma$  but the mere addition of one more unit cell provides transmittivities of 60% in the optical. Finally  $\Gamma$  the filter is completely opaque for any  $\omega < \gamma$ . As the air (dielectric) thickness increases  $\Gamma$  the passbands can be as densely distributed as desired with corresponding decrease of their bandwidth.

## References

- [1] See C. Bohren and D. Huffman  $\Gamma$  *Absorption and Scattering of Light by Small Particles*  $\Gamma$  John Wiley & Sons  $\Gamma$  Inc.  $\Gamma$  New York  $\Gamma$  1983  $\Gamma$  pp. 251-267 and references therein.
- [2] M. Scalora et al.  $\Gamma$  "Transparent  $\Gamma$  Metallo-Dielectric  $\Gamma$  One-Dimensional  $\Gamma$  Photonic band gap Structures"  $\Gamma$  (Sept. 15  $\Gamma$  1997)  $\Gamma$  to be published in Jour. App. Phys.
- [3] These formulas are generalizations of the ones for normal incidence derived in H. Contopanagos  $\Gamma$  N. Alexopoulos and E. Yablonovitch  $\Gamma$  "High-Q Radio Frequency Structures using One-Dimensionally Periodic Metallic Films"  $\Gamma$  (June 6  $\Gamma$  1997)  $\Gamma$  to be published in IEEE Trans. MTT.