High-Q Rectangular Cavities and Waveguide Filters using Periodic Metalo-Dielectric Slabs

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Abstract

We obtain the exact solution of Maxwell's equations for the fields in rectangular waveguide or cavity loaded with metalo-dielectric slabs of arbitrary thicknesses and permitivities, placed periodically along the longitudinal direction. For cavity applications, we find significant Q-factor enhancements in the microwave regime, relative to cavities made of bulk conductor walls. For waveguide filters, the system allows frequency multiplexing with very competitive filter lengths.

Introduction

Good conductors are excellent reflectors at microwave frequencies. An important issue is whether it is possible to design a metalo-dielectric periodic structure of reflectivity higher than the intrinsic reflectivity of the bulk metal itself. It is well known that, if a dielectric material of modest intrinsic reflectivity is considered, one can design a multilayer structure of dielectric films whose reflectivity is far greater than the intrinsic reflectivity of the bulk dielectric[1]. The purpose of this paper is twofold:

First, examine whether this is true for metals, whose reflectivity at microwave frequencies is very high to start with, even for single metallic films of thicknesses much smaller than the skin depth[2] and determine the parameters for maximum reflectivity.

Secondly, examine the performance of practical realizations of these structures, i.e., metallic films backed by dielectrics, placed within a rectangular cavity or waveguide, for maximum Q-enhancement and filter performance, respectively. To back the metal films, we will be using high-permitivitty, low-loss dielectrics, which provide high reflectivity in a variety of specific geometries, different than ours, treated in the literature[3]-[5].

Design equations

Consider the rectangular waveguide of Fig.1, where the filter shown has been placed. We will denote with $\{d_c,\ d_d,\ d_a\}$ the thicknesses of the conducting film, dielectric slab, and air gap in the filter's unit cell and our filter will contain an arbitrary number N of such unit cells. We assume that the sidewalls are perfect electric conductors, and have solved Maxwell's equations for all the modes supported. Because of the symmetry of the problem, mode classification is the same as in

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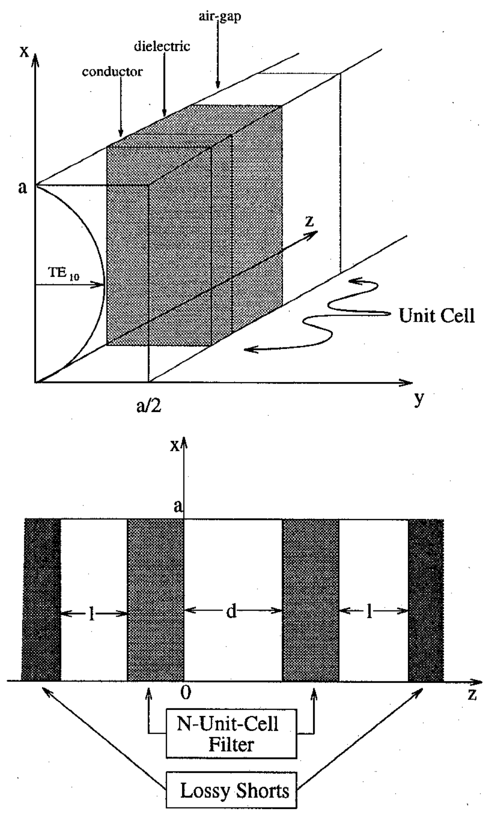

Figure 1: TE_{10}-mode scattering off the N-Unit-Cell Filter and corresponding cavity.

the empty waveguide and the solutions can be interpreted as describing exact scattering of normal waveguide modes off the filter. Here, we will present only our solution for the dominant TE_{10} mode.

We find the reflection coefficient for an open-ended waveguide to be[6]

$$\Gamma_N^{1,0}(\theta) = 2f_{21}\left[(1+\zeta)^N - (1-\zeta)^N\right] \times \left\{(f_{11}-f_{22})\left[(1+\zeta)^N - (1-\zeta)^N\right] + \zeta(f_{11}+f_{22})\left[(1+\zeta)^N + (1-\zeta)^N\right]\right\}^{-1} \quad (1)$$

where

$$\zeta = \frac{f_{11}-f_{22}}{f_{11}+f_{22}}\sqrt{1+\frac{4f_{12}f_{21}}{(f_{11}-f_{22})^2}} \quad . \quad (2)$$

The matrix elements f_{ij} are obtained from the transmission matrix through one unit cell of the filter:

$$f_{11} = 1 - \Gamma^1_{a,c}\Gamma^1_{a,d}\ ,\ f_{21} = \Gamma^1_{a,c} + \Gamma^1_{a,d}Z_{a,c}$$
$$f_{12} = -\mathrm{e}^{-2\gamma_a d_a}(\Gamma^1_{a,d} + \Gamma^1_{a,c}Z_{a,d})$$
$$f_{22} = \mathrm{e}^{-2\gamma_a d_a}(-\Gamma^1_{a,c}\Gamma^1_{a,d} + Z_{a,c}Z_{a,d}). \quad (3)$$

where the functions $\Gamma^1_{a,j}$ are the reflection coefficients of 1 slab of material $j \in \{\text{conductor} \equiv c,\ \text{dielectric} \equiv d\}$ for perpendicularly-polarized plane-wave oblique incidence from air

$$\Gamma^1_{a,j} = \Gamma_{a,j}\left(\frac{1-\mathrm{e}^{-2\gamma_j d_j}}{1-(\Gamma_{a,j})^2\mathrm{e}^{-2\gamma_j d_j}}\right) \quad , \quad (4)$$

and $Z_{a,j}$ are closely related (but irreducible) functions

$$Z_{a,j} = \frac{-(\Gamma_{a,j})^2 + \mathrm{e}^{-2\gamma_j d_j}}{1-(\Gamma_{a,j})^2\mathrm{e}^{-2\gamma_j d_j}} \quad . \quad (5)$$

In these equations appear the intrinsic (bulk) reflection coefficients of the materials, $\Gamma_{a,j}$, and propagation constants, γ_j, for the above excitation:

$$\Gamma_{a,j} = \frac{(\eta_j/\cos\theta_j) - (1/\cos\theta)}{(\eta_j/\cos\theta_j) + (1/\cos\theta)},$$
$$\gamma_a = jk_0\cos\theta,\ \gamma_j = jk_0\cos\theta_j/\eta_j\ , \quad (6)$$

where η_j are the relative wave impedances

$$\eta_c = \frac{1}{\sqrt{1-j\frac{\sigma}{\omega\epsilon_0}}}\ ,\ \eta_d = \frac{1}{\sqrt{\epsilon}\sqrt{1-j\alpha}} \quad . \quad (7)$$

and σ, ϵ, α are the conductivity of the conductor, the real part of the relative permitivity and the loss tangent of the dielectric, respectively. The waveguide dispersion properties are introduced through the "angular dependence" θ:

$$\sin\theta = \frac{1}{k_0}\left(\frac{\pi}{a}\right) \ , \ \cos\theta_j = \sqrt{1-\eta_j^2 \sin^2\theta}\ . \tag{8}$$

The transmission coefficient through the filter, $T_N^{1,0}(\theta)$, can also be similarly calculated.

If the waveguide is shorted at one end with bulk metal of conductivity σ' after insertion of the filter, or if we consider a cavity produced by closing both ends of the waveguide in the same manner, as indicated in Fig. 1, then the reflection coefficient off the composite wall (Filter+Short) is

$$\Gamma_{N+S}^{1,0}(\theta) = \frac{\Gamma_N^{1,0} + B_N^{1,0} e^{-2\gamma_a(l-d_a)} \Gamma_{a,S}}{1+\Gamma_N^{1,0}\left(\frac{f_{12}}{f_{21}}\right) e^{-2\gamma_a(l-d_a)}\Gamma_{a,S}} \ , \tag{9}$$

where $\Gamma_{a,S}$ is given by the same formula as $\Gamma_{a,c}$ if we substitute for the conductivity of the short, σ', and

$$B_N^{1,0} = \left\{ -(f_{11}-f_{22})\left[(1+\zeta)^N - (1-\zeta)^N\right] + \zeta(f_{11}+f_{22})\left[(1+\zeta)^N + (1-\zeta)^N\right] \right\} \times \left\{ (f_{11}-f_{22})\left[(1+\zeta)^N - (1-\zeta)^N\right] + \zeta(f_{11}+f_{22})\left[(1+\zeta)^N + (1-\zeta)^N\right] \right\}^{-1} . \tag{10}$$

All the above equations can trivially produce simplified systems. If we set $\theta = 0$ we effectively remove the waveguide ($a \to \infty$) and obtain formulas for plain wave normal incidence. The corresponding cavity would be a Fabry-Perot Resonator with planar walls. If θ is treated as a frequency-independent input angle, oblique TE incidence is described. The short is removed if we set $\Gamma_{a,S} = 0$, while any material layer j is removed from the filter by setting the corresponding thickness d_j to zero,

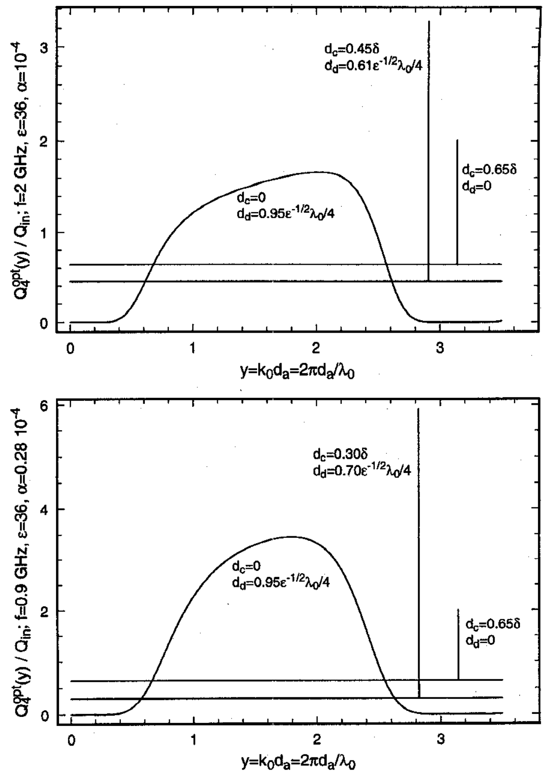

Figure 2: Optimum Q (N=4) of metalo-dielectric filters for normal plane-wave incidence.

and the resulting formulas have a smooth analytical and numerical limit.

A sample of results

In Fig. 2 we show the unloaded Q-factor (finesse), $Q_N = (1-|\Gamma_N(\theta=0)|^2)^{-1}$, for normal plane-wave incidence, for a metallic, dielectric and metalo-dielectric systems of optimized thicknesses for maximum reflectivities. We have normalized to the corresponding intrinsic Q-factor of bulk metal (Q_{in}), which is equal to 11,945 ($f = 0.9$ GHz) and 8,012 ($f = 2$ GHz) for Copper, and δ is the conductor skin depth. We are using $BaTiO_3$

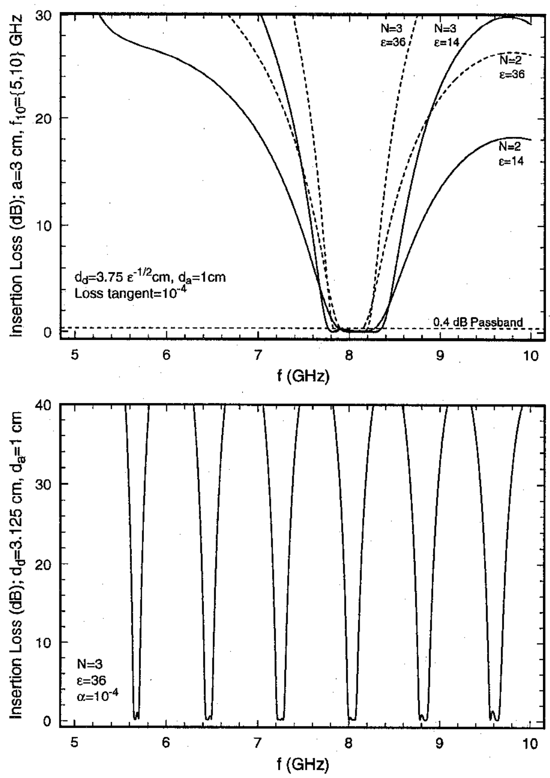

Figure 3: Frequency multiplexing of dielectric waveguide filter for TE_{10} excitation.

as a dielectric, with $\epsilon = 36$, $\alpha = 0.28 \times 10^{-4}$ (0.9 GHz), 10^{-4} (2 GHz).

In Fig. 3 we show the dielectric waveguide filter response for a long side $a = 3$ cm, for different permitivities. The single-channel filter has a total length of $1.625N$-1 cm, while the six-channel multiplexer has a total length of $4.125N$-1 cm.

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