

LIGHT EMISSION IN PHOTONIC CRYSTAL MICRO-CAVITIES

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INTRODUCTION:

The creation of light has pre-occupied mankind, probably ever since cave men came to control fire. Since the invention of the electric light bulb, there have already been several revolutions in our ability to control light. One of these has been stimulated emission which gave rise to lasers. There have been many other developments recently which involve control over amplitude and phase of electromagnetic waves, including for example photon number state squeezing¹.

At the present time in opto-electronics, we are in the midst of yet another revolution in the artificial creation of light. This comes about due the recognition of the role of zero-point electromagnetic field fluctuations in the light emission process. We now recognize that in electromagnetic micro-cavities², and in other peculiar³ local field geometry's, zero point electromagnetic fields can be engineered and modified to serve our needs. In particular, we can anticipate that in electromagnetic micro-cavities the dipole matrix element for light emission, which is proportional to the zero-point electric field, can become stronger than the damping. In those circumstances, the electromagnetic interaction with matter becomes non-perturbative, and can best be described by the new⁴ concept of "cavity polaritons". This new form of light generation is neither stimulated emission, nor spontaneous emission, but a new strongly coupled, non-perturbative, regime in which the light emission process can be extra-ordinarily fast. Cavity polaritons may well be the next frontier in mankind's ability to create and control light. These ideas will be discussed by Weisbuch as well as others among the lectures in this Volume.

In this lecture, we will focus on the fundamental statistical mechanics of those opto-electronically active materials which are able to store energy internally and then emit it as light. Following that, we will analyze different viewpoints of spontaneous emission, particularly as influenced by zero-point electric fields. Then we will introduce doped "Photonic Crystal" micro-cavities, 3-dimensional nanostructures which will provide us with the ultimate control over zero-point electromagnetic field fluctuations.

OPTO-ELECTRONICALLY ACTIVE MATERIALS:

When light is absorbed by a material, the photon energy is usually converted almost immediately into heat. For the vast array of materials and substances in nature, and most of those made by man, electronic energy is not effectively internally stored and is dissipated away as heat. We regard such materials as opto-electronically *inactive*. They do not store the energy long enough to emit light efficiently.

In contrast, there are a few categories of condensed phase materials, such as semiconductors, organic dyes, and rare-earth or transition metal doped crystals, which store electronic energy internally. Among these few categories of materials, non-radiative thermalization is slow enough to allow most of the energy to be emitted as light. These are the precious opto-electronically *active* materials which are the basis of much industry and the science of this volume. (They should not be confused with the many substances which are simply colored, absorbing light selectively at various wave-lengths, but which do not store electronic energy.)

The opto-electronically active materials have an elegantly simple statistical mechanics, which is helpful in analyzing the operation of light emitting and absorbing devices such as lasers, light-emitting-diodes, and solar cells. The internal energy storage is symbolized by a Free Energy, μ , which is the change of internal free energy in the condensed phase material by the addition of a single electronic excitation, as might be produced by the absorption of a single photon. This Free Energy has a simple interpretation for both semiconductors and dyes as illustrated in Figure 1:

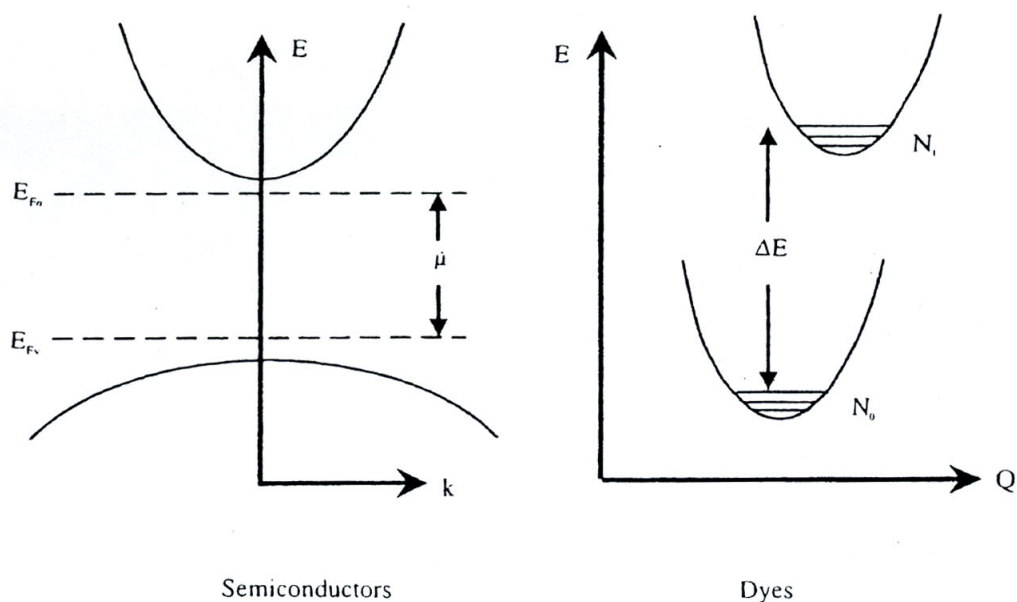


Figure 1: The energy level structure of a direct gap semiconductor and an organic dye molecule. For semiconductors, energy E is plotted versus carrier wave vector, k , and for dye molecules energy is plotted against configuration co-ordinate Q . Both substances store energy internally as quantified by the internal chemical potential, or free energy, μ . In semiconductors we recognize that μ equals the separation of electron and hole quasi-Fermi levels $E_{Fn} - E_{Fv}$.

For the unexcited semiconductor in the dark, the two quasi-Fermi levels coincide at the usual equilibrium Fermi level. Upon excitation of carrier density above the normal background densities, a non-zero chemical potential is developed; $\mu = E_{Fc} - E_{Fv}$.

In dye molecules there are no Fermi-Dirac statistics, since in effect, the electron and the hole are not free, but are constrained to sit together on one molecular site. Then Boltzmann statistics must be used to describe the internal chemical potential: $\mu = \Delta E + kT \ln(N_1 / N_0)$, where ΔE is the energy difference between excited and ground states, and N_1 and N_0 are the respective populations. Of course in Boltzmann equilibrium (in the dark for example), the two terms exactly cancel leaving the chemical potential $\mu=0$.

The essence of an opto-electronically active material is that there are two reservoirs, for example the conduction band reservoir and the valence band reservoir in quasi-equilibrium. Within each reservoir there is rapid thermalization to the ambient temperature. But between upper and lower reservoirs there is only slow equilibration and energy storage is practical. Presently, we will see that it is possible to establish a further quasi-equilibrium between an excited opto-electronically active material and a surrounding Planck-like radiation field.

THERMODYNAMICS OF OPTO-ELECTRONIC MATERIALS:

It is well known that electromagnetic radiation carries entropy. In fact, the photon carries energy $h\nu$, as well as⁵ an entropy ΔS , and a free energy. In thermal equilibrium, the Planck formula gives a Brightness B :

$$B(\nu, T) = \frac{8\pi n^2 \nu^2}{c^2} \frac{1}{e^{h\nu/kT} - 1} \quad (1)$$

where B is in photons/area, per unit bandwidth, per unit time, per 4π steradians, n is the refractive index, and kT the thermal energy. Assuming the photon gas is an effective heat bath, the addition of 1 photon of energy $h\nu$ is accompanied by an entropy change $h\nu/T$ which can be calculated by inverting equation (1):

$$\Delta S = \frac{h\nu}{T} = k \ln \left(1 + \frac{8\pi n^2 \nu^2}{c^2 B} \right) \quad (2)$$

Equation (2) may be regarded as the entropy of 1 photon which depends on the brightness B of the radiation field. At the same time we may calculate the entropy change of an opto-electronic material with internal free energy $\mu = h\nu - T\Delta S$ is:

$$\Delta S = \frac{h\nu - \mu}{T} \quad (3)$$

where $h\nu - \mu$ is the energy being thermalized. Equation (3) and equation (2) are the entropy change of the material system and radiation field respectively, upon the transfer of a single photon from one to the other. Quasi-equilibrium is established when the overall entropy change is at an extremum which requires that the two entropies must balance which occurs at a Brightness:

$$B(\nu, \mu, T) = \frac{8\pi n^2 \nu^2}{c^2} \frac{1}{e^{(h\nu - \mu)/kT} - 1} \quad (4)$$

Equation (4) is a Generalized Planck formula for radiation fields which are in quasi-equilibrium with excited opto-electronic materials storing an internal free energy μ . It differs from the usual Planck formula in that most materials sustain no internal free energy, $\mu=0$.

Given that quasi-equilibrium can be achieved, we can invoke the Principle of Detailed Balancing which requires that each physical process be exactly balanced by the reverse process. In this case Luminescence, L , or spontaneous emission into the quasi-equilibrium radiation field is balanced exactly by absorption of photons from the radiation field:

$$L(\nu, \mu, T) = \alpha(\nu, \mu, T) B(\nu, \mu, T) \quad (5)$$

where $\alpha(\nu, \mu, T)$ is the absorption co-efficient in cm^{-1} , which depends explicitly on frequency ν , ambient temperature T , and material excitation μ . Substituting the quasi-equilibrium brightness, eq'n. (4), into the detail balanced luminescence, eq'n. (5), we have a quantitative expression for the expected spontaneous emission from excited optoelectronic materials:

$$L(\nu, \mu, T) = \frac{8\pi n^2 \nu^2}{c^2} \frac{\alpha(\nu, \mu, T)}{e^{(h\nu - \mu)/kT} - 1} \quad (6)$$

When the material is only moderately excited, $\mu \ll h\nu$, the absorption spectrum is unchanged from that of unexcited material $\alpha(\nu, \mu, T) \equiv \alpha(\nu, 0, T)$ and the 1 in the denominator is often dropped as being negligible. Then the spontaneous emission spectrum assumes the following simple form:

$$L(\nu, \mu, T) = \exp\left\{\frac{\mu}{kT}\right\} \times \frac{8\pi n^2 \nu^2}{c^2} \alpha(\nu, 0, T) \exp\left\{-\frac{h\nu}{kT}\right\} \quad (7)$$

This relationship is illustrated by the following graph of both the absorption spectrum and luminescence spectrum:

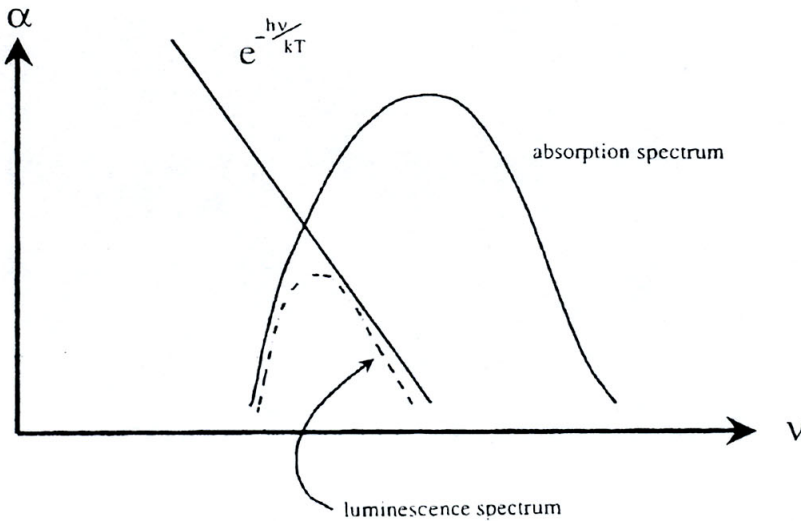


Figure 2: The detailed balancing relationship between the absorption co-efficient, α , the Luminescence L , and the Boltzmann factor according to equation (7).

Equation (7) says that the luminescence spectrum is simply the absorption spectrum multiplied by a Boltzmann factor of the photon energy. This is a rather famous formula which has been rediscovered many times in the history of physics in many contexts. It is a

generalization of Einstein's A and B co-efficients to case of energy bands in condensed matter. Indeed it was first discovered independently of Einstein by the American theorist⁶ E. H. Kennard in (1918) for the case of dye molecules. It was independently re-discovered⁷ in 1954 by Shockley and von Roosbroeck for the case of semiconductors where eq'n. (7) is called the Shockley-von Roosbroeck relation. It's application to semiconductors is nicely outlined by Pankove⁸ who shows how it correctly predicts even the absolute luminescence spectrum of indirect-gap semiconductors such as Germanium. We have followed rather closely the elegant derivation given by Ross⁹ in the context of the thermodynamics of photosynthesis in plant chlorophyll.

There is one final insight to be derived from this fertile thermodynamic analysis. Let us invert the exact Detailed Balancing expression eq'n. (6), which is valid even when $\mu \geq h\nu$, giving the absorption co-efficient as a product of the Luminescence spectrum and a Planck-like factor:

$$\alpha(\nu, \mu, T) = \frac{c^2}{8\pi n^2 \nu^2} \overbrace{L(\nu, \mu, T)}^{\text{positive}} \overbrace{\left[\exp\left\{\frac{h\nu - \mu}{kT}\right\} - 1 \right]}^{\text{negative}} \quad (8)$$

Note that Luminescence is an energy emission which can only be positive, but that the Planck-like factor; $[\exp\{(h\nu - \mu)/kT\} - 1]$, can become negative for $\mu > h\nu$. The product of a positive number and a negative number is negative, implying negative absorption, α , or in other words gain! Although the thermodynamic derivation is not meant to be applicable so far from quasi-equilibrium, eq'n. (8) predicts gain and lasing.

The transparency condition at which gain occurs $\alpha \leq 0$ is $h\nu \leq \mu$, or in words, the internal chemical potential must exceed the photon energy. In semiconductors this is famous¹⁰ as the Bernard-Duraffourg condition. But we see that it applies equally well to dye molecules.

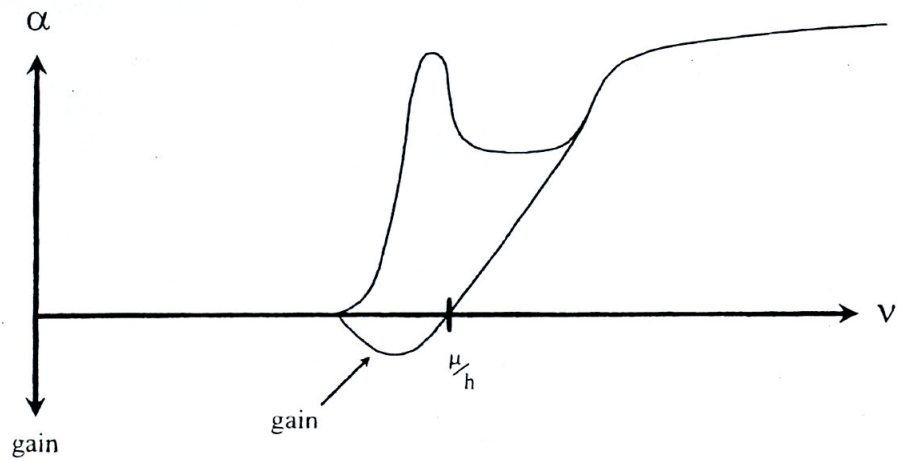


Figure 3: Typical absorption spectra of both an unexcited opto-electronic material and the corresponding excited material in which the chemical potential begins to overlap the absorption edge. A small band of gain occurs just at the onset of the absorption spectrum.

SPONTANEOUS EMISSION RATE BY FOUR METHODS

In free space it is actually quite easy to calculate the expected rate of spontaneous emission from an excited material. However, in the real geometry of a nano-structured opto-electronic device it is actually quite difficult to quantize the electromagnetic modes, or indeed even to calculate the electromagnetic modes. As a result we have developed a number of approximate methods for calculating spontaneous emission, which is really an admission that no method actually meets all our needs.

Method 1. Detailed Balancing

This method is really equivalent to that of eq'n. (7), except that we will now express the absorption in terms of a matrix element squared and a density of electron states by employing Fermi's Golden Rule:

$$\alpha = \frac{4\pi^2}{cn} \omega \underbrace{|ex_{ij}|^2}_{\text{dipole matrix element}} \times \underbrace{\frac{dN}{dE}}_{\text{electron density of states}} \quad (9)$$

where the density of final states is the density of upper electron levels and $x_{ij} \equiv \langle i|x|j \rangle$ is the matrix element. In the manner with which we have become familiar, eq'n. (9) will now be multiplied by the black-body spectrum containing $\exp\{-h\nu/kT\}$ in the Planck limit, which will give us the statistical mechanical probability of occupation of those upper electron levels. The product of density of electron states times $\exp\{-h\nu/kT\}$ is the number of excited levels, so that dropping those terms gives the detailed balance spontaneous emission rate normalized per excited state:

$$L = \frac{4}{3} \frac{|ex_{ij}|^2 \omega^3 n}{\hbar c^3} \quad (10)$$

which is the conventional spontaneous emission rate except that the refractive index n , appears here explicitly.

Method 2. Second Quantization

In this approach we once again use Fermi's Golden Rule, but the electromagnetic field is now quantized per Dirac's recipe, and the density of final states is now the density of optical modes into which the photon can be emitted. The matrix element contains an electric field operator which contains a^+ the photon number raising operator which adds a spontaneously emitted photon to the zero point state:

$$\hat{\mathcal{E}} = \frac{1}{n} \left(\frac{2\pi\hbar\omega}{V} \right)^{\frac{1}{2}} e^{ik \cdot r} (a^+ + a) \quad (11)$$

where V is the normalization volume for the electromagnetic modes. Then Fermi's Golden rule then looks as follows:

$$\frac{2\pi}{\hbar} \underbrace{\left| \langle 1, j | ex \cdot \hat{\mathcal{E}}^+ | 0, i \rangle \right|^2}_{|ex_{ij}|^2 \left| \langle 1 | \hat{\mathcal{E}}^+ | 0 \rangle \right|^2} \times \underbrace{\frac{n^3 \omega^2 V}{\pi^2 \hbar c^3}}_{\text{optical density of states}} \quad (12)$$

In equation (12), the matrix element was factored into a material part and an electromagnetic field part. Collecting terms, the spontaneous emission rate is:

$$\text{Transition Rate} = \frac{4}{3} \frac{|ex_{ij}|^2 n \omega^3}{\hbar c^3} \quad (13)$$

which is the same as eq'n. (10), not surprisingly.

Method 3. Matter Consists of Radiating Dipoles

Suppose that we model the spontaneous emission as being due to accelerating charge inside the excited material.

$$\left. \begin{array}{l} \text{Power Radiated} \\ \text{by accelerating charge} \end{array} \right\} = \frac{2}{3} \frac{ne^2 v^2}{c^3} \quad (14)$$

where v is the electron acceleration and e is the electron charge, and we have included the effect of a medium of refractive index n . Converting from radiated power to transition rate by dividing by $\hbar\omega$:

$$\text{Photon Emission rate} = \frac{2}{3} \frac{n \omega^3 |x|^2}{\hbar c^3} \quad (15)$$

where the acceleration has been accounted for by bringing down two powers of ω and $|x|^2 \equiv \langle j|x^2|j \rangle$ is the mean square motion of the electron in the excited state. It remains then to calculate the expectation value of the mean square electron position fluctuations, $\langle j|x^2|j \rangle = \langle j|x \cdot x|j \rangle$. Inserting a complete set of states, i , in place of the dot product, the mean square fluctuations become:

$$\langle j|x^2|j \rangle = \sum_i \langle j|x|i \rangle \langle i|x|j \rangle = \sum_i \langle j|x|i \rangle^2 = |x_{ij}|^2 \quad (16)$$

where it is approximated that the summation is dominated by the coupling of the excited state j to only a single state, the ground state, i . Likewise a similar calculation in the ground state gives the same result: $\langle i|x^2|i \rangle = |x_{ij}|^2$. Thus we recognize that eq'n. (15), the spontaneous emission due to the acceleration of electrons inside atoms gives half the usual amount of spontaneous emission. Furthermore, since ground states have equal dipole fluctuations as excited states, paradoxically, they too would radiate. This may be regarded as spontaneous emission due to the zero fluctuations of matter, which are present even in the ground state! We will find a way to stabilize ground states shortly.

Method 4. Spontaneous Emission Due to Electromagnetic Zero-Point Fluctuations

In this approach we consider the zero-point fluctuations (ZPF) semi-classically and we ask what happens when they fall on an absorbing medium. Naturally, the ZPF would be absorbed, and this absorption would have to be compensated by spontaneous emission. In other words there is detailed balancing not only of thermal radiation, but also of ZPF. Since the intensity, I of zero-point electromagnetic waves is $c/n \times$ energy density, it may be written in terms of one-half quantum of energy, in a normalization volume V , and a density of states in an energy bandwidth dE :

$$I = \frac{c}{n} \times \frac{\hbar\omega}{2V} \frac{n^3 \omega^2 V}{\pi^2 \hbar c^3} dE \quad (17)$$

combining all factors the ZPF intensity is:

$$I = \frac{n^2 \omega^3}{2\pi^2 c^2} dE \quad (18)$$

When the ZPF are multiplied by absorption co-efficient, eq'n. (9), the result is a type of semiclassical absorption rate:

$$\text{energy absorption rate} = \frac{2}{3} \frac{n |\mathbf{ex}_{ij}|^2 \omega^4}{c^3} \quad (19)$$

which must be balanced by a photon emission rate:

$$\text{photons per unit time} = \frac{2}{3} \frac{n |\mathbf{ex}_{ij}|^2 \omega^3}{\hbar c^3} \quad (20)$$

which is the same as eq'n. (15), (method 3) and half the rate given by eq'ns. (10) and (13), (methods 1 & 2). Likewise we note that if the material system had been in the excited state, the ZPF would induce stimulated emission rather than absorption, again at only half the rate needed to explain. Thus we have a somewhat paradoxical situation which was first resolved¹¹ by Cohen-Tannoudji and co-workers:

Method 3 and 4 each give half the expected rate of spontaneous emission from excited states, due to dipole fluctuations of matter, and due to "stimulated emission" induced by ZPF on the excited state, respectively. If we add the spontaneous emission from both methods 3 and 4, then we obtain the correct, full amount of spontaneous emission. Likewise, by method 3, the ground state paradoxically radiates, but by method 4 it absorbs ZPF, exactly balancing that radiation. This method of stabilizing ground states was first introduced by Fain¹². The overall balance of spontaneous emission, stimulated emission, and ZPF is illustrated by Figure 4.

The stabilization can also be regarded as another version of Kirchoff's law which says that a partially absorbing gas in thermal must have enough thermal fluctuations to exactly compensate the absorption of black-body radiation. We need only note that the half quantum of ZPF are often regarded as part of the Bose Einstein occupation probability. In that case the stabilization of ground states may be simply regarded as a generalized version of Kirchoff's law as applied to ZPF.

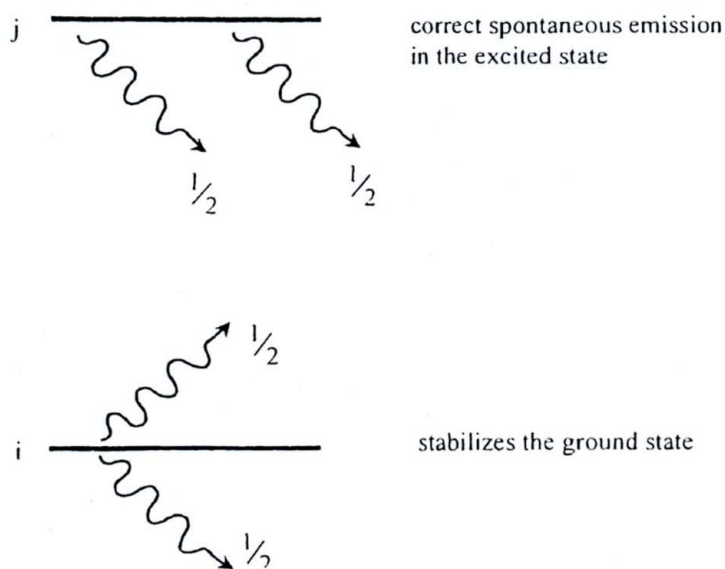


Figure 4: The dipole fluctuations of matter, and the ZPF of the electromagnetic field, each contribute at half the expected rate, to produce the correct spontaneous emission from the excited state j. But the matter and field zero point fluctuations cancel in the ground state i, explaining why ground states are stable.

THE EFFECT OF GEOMETRY ON SPONTANEOUS EMISSION

We see from the need for 4 different methods for calculating spontaneous emission rates, that in most practical geometries the calculation is actually somewhat difficult. In realistic geometries, which are inevitably complex, it is not possible to solve for every electromagnetic mode. Indeed one of the key problems for spontaneous emission calculations is that every mode, both near and far from the material system, may in principle accept spontaneous emission. This is sometimes forgotten, particularly for planar cavity structures which inevitably contain wave-guide modes having nothing to do with the high-Q cavity mode. These wave-guide modes generally accept most of the spontaneous emission and it is erroneous to neglect them. For our purposes we will select from among the 4 different calculational methods whichever one is most convenient for a given geometry.

As an example, let us calculate the spontaneous emission from a small spherical particle, much smaller than an optical wavelength, of dielectric constant ϵ_{int} surrounded by an infinite medium of dielectric constant ϵ_{ext} . Recognizing that the particle is bathed in zero point fluctuations coming from infinity, let us employ method 4. As is well-known, the local field \mathcal{E}_{int} inside a spherical particle differs from the incident field, \mathcal{E}_{ext} of the ZPF due to the local field correction factor. In the spherical case the local field correction factor is:

$$\mathcal{E}_{\text{int}} = \frac{3\mathcal{E}_{\text{ext}}}{\left(\frac{\epsilon_{\text{int}}}{\epsilon_{\text{ext}}}\right) + 2} \quad (21)$$

where $\epsilon_{\text{int}} = n_{\text{int}}^2$ the refractive index squared, and $\epsilon_{\text{ext}} = n_{\text{ext}}^2$. We will regard the condition of a matched refractive index $n_{\text{ext}} = n_{\text{int}} = n_{\text{matched}}$ be the reference case, i.e. all spontaneous emission lifetimes will be referenced to that case. Then the absorbed ZPF referenced to the matched case is:

$$\frac{\alpha I_{\text{int}}}{\alpha I_{\text{matched}}} = \frac{n_{\text{int}}}{n_{\text{ext}}} \left[\frac{3}{\left(\frac{n_{\text{int}}^2}{n_{\text{ext}}^2}\right) + 2} \right]^2 \frac{I_{\text{ext}}}{I_{\text{matched}}} \quad (22)$$

$\underbrace{\qquad\qquad\qquad}_{\frac{n_{\text{ext}}^2}{n_{\text{int}}^2}}$

where α is the absorption coefficient, and we make explicit note that the ratio $I_{\text{ext}}/I_{\text{matched}}$ is $n_{\text{ext}}^2/n_{\text{int}}^2$ as indicated by eq'n. (18). Simplifying eq'n. (22), and letting the particle have a typical semiconductor refractive index $n_{\text{int}} = 3.5$ immersed in vacuum $n_{\text{ext}} = 1$:

$$\frac{\text{spont. em. rate}}{\text{matched sp. em. rate}} = \frac{n_{\text{ext}}}{n_{\text{int}}} \left[\frac{3}{\left(\frac{n_{\text{int}}^2}{n_{\text{ext}}^2}\right) + 2} \right]^2 \approx \frac{1}{80} \quad (23)$$

Thus the spontaneous emission is slowed by a substantial factor merely³ by immersion of the spherical speck in refractive index unity.

There are a number of additional cases which were treated by S. T. Ho et al¹³. For example in thin film geometry, the influence on spontaneous emission rates is different for TE and TM electromagnetic waves. For the TE case, the effect is relatively modest:

$$\text{TE case: } \frac{\text{spont. rate}}{\text{matched rate}} = \left(\frac{n_{\text{ext}}}{n_{\text{int}}} \right) \quad (24)$$

However for the TM case of electromagnetic waves perpendicular to the thin film,

$$\text{TM case: } \frac{\text{spont. rate}}{\text{matched rate}} = \left(\frac{n_{\text{ext}}}{n_{\text{int}}} \right)^5 \sim \frac{1}{525} \quad (25)$$

the effect can be huge. Likewise for each possible geometry including;

1. specks¹⁴ of size one-half optical wavelength,
2. disks¹⁵ of diameter several wavelengths, (whispering gallery cavities),
3. broad area¹⁶ vertical cavity structures,
4. narrow post¹⁷ vertical cavity structures,

it becomes necessary to implement a detailed calculation which includes not only the obvious modes, such as the cavity modes, but also the less obvious leakage modes which send spontaneous emission into a sink at infinity. This is discussed at length in the contributions by Bjork and Yokoyama in this Volume. One of the most interesting cavities from this point of view is the doped photonic crystal. By doping, we mean the intentional introduction of a defect in an otherwise perfectly periodic 3-dimensional dielectric structure as discussed in ref. 2. Photonic crystals are reviewed in a special issue¹⁸ of the Journal of the Optical Society, and a recent review paper is reproduced here. (See also the workshop proceedings edited by Soukoulis, and the reviews in the Journal of Modern Optics).

MICRO-CAVITY FIGURE OF MERIT

There are two parameters in which micro-cavities have an advantage over conventional large laser cavities. The first parameter is the speed of spontaneous emission as given by Fermi's Golden Rule, which appears as follows in a micro-cavity. The rate of spontaneous emission is:

$$\frac{2\pi}{\hbar} \left| \langle \text{ex} \cdot \hat{\mathcal{E}}^+ \rangle \right|^2 \frac{1}{\hbar \Delta \omega} \quad (26)$$

where the density of states is one (for a single mode cavity), per cavity line-width, $\Delta \omega$. Given that the material dipole moment $\langle \text{ex} \rangle$ is regarded as fixed, the spontaneous emission rate can be increased by either improving the cavity Q, or by increasing the strength of the electric field operator \mathcal{E}^+ , whose magnitude is equivalent to the root mean square zero-point electric field.

Improving the cavity Q has its limits since there is not much point in having a cavity linewidth which is much narrower than the material linewidth. In such a case, the poor spectral overlap would force a point of diminishing returns when the cavity Q is as good as the material Q. The real leverage in eq'n. (26) is in the zero point electric field \mathcal{E} . The electric field is determined by the energy density:

$$\epsilon_{\text{int}} \frac{\mathcal{E}^2}{4\pi} = \frac{\hbar \omega}{2} \times \frac{1}{V_{\text{cav}}} \quad (27)$$

which is one-half quantum within the cavity volume V_{cav} . Clearly the major leverage is in reducing V_{cav} , and hence we seek microscopic cavities of the smallest volume possible.

Doped photonic crystal micro-cavities confine the electromagnetic radiation in all 3 dimensions yielding the smallest possible electromagnetic cavity volumes. This can influence the rate of spontaneous emission, eq'n. (26), which is of considerable technical importance in regard to the desire for very high modulation speeds for communications purposes using light emitting diodes and lasers. If this increase in spontaneous emission rate is carried to the ultimate degree, a strong \mathcal{E} can help achieve a large enough dipole matrix element to allow us to enter the cavity polariton regime⁴ in which $|ex\mathcal{E}|$ is stronger than the cavity damping $\Delta\omega$. Then the time dependent perturbation theory of Fermi's Golden Rule breaks down and we enter a coherent regime of very fast conversion between electronic energy and electromagnetic energy.

The second Figure of Merit is the cavity β which is the fraction of spontaneous emission which is captured by the cavity mode. This is important for photon number state squeezing¹. The rate of spontaneous emission is proportional to the energy density of zero point fluctuations. For the cavity mode this is represented by the fraction:

$$\beta \sim \text{cavity Z.P.F.} = \frac{\hbar\omega}{2} \frac{1}{V_{\text{cav}}} \quad (28)$$

While for all the background modes this fraction is represented by:

$$1-\beta \sim \text{background Z.P.F.} = \frac{\hbar\omega}{2} \frac{n^3\omega^2\Delta\omega}{\pi^2c^3} \quad (29)$$

In the case of a doped photonic crystal cavity, the electromagnetic mode is totally confined and there is no background continuum for that case. The Figure of Merit is the following ratio which should be as small as possible, which for open cavities is:

$$\frac{1-\beta}{\beta} = \frac{8\pi V_{\text{cav}}}{\left(\frac{\lambda}{n}\right)^3 Q} \quad (30)$$

which can be re-written as:

$$\beta = \frac{1}{\frac{8\pi V_{\text{cav}}}{\left(\frac{\lambda}{n}\right)^3 Q} + 1} \quad (31)$$

where λ is the free space wavelength. The smallest practical cavity volume is probably $V_{\text{cav}}=(\lambda/2n)^3$, while $Q\sim 20$ may be a practical material Q for semiconductors at room temperature. This would limit the fraction of spontaneous emission which can be captured by the cavity mode to ≈ 0.85

The applications toward lower threshold currents, cavity polaritons, and single-mode light-emitting diodes exhibiting laser-like performance will continue to push this field to smaller and smaller cavity volumes.

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