

# The thermodynamic limits of light concentrators

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Much work has been done over the past 10 years on the concentration of solar radiation using a variety of devices. To concentrate the light, photons from a larger area are collected and directed to a smaller area. Some devices use geometrical optics, or a change in index of refraction to increase the illumination on a surface above the incident solar level. Other systems use a frequency or Stokes shift to increase the illumination of light at one photon energy at the expense of another. There is often confusion as to the concentration limit imposed by basic physical laws on these different types of systems. Presented is a unification of the ideas and principles developed for the various classifications of concentrators. Equations are developed that describe the limits of concentration in geometrical and fluorescent systems. Concentration is shown to be a function of the index of refraction, angular collection range, as well as the frequency shift. Applications of the ideas involve the understanding of diffuse radiation concentrators and the solar powered laser.

## 1. Introduction

Almost everyone has had experience with a light concentrator. A hand lens focused on the ground on a sunny day demonstrates the ease with which sunlight can be collected and concentrated. What is not so well accepted are the mechanisms for this concentration or the limits imposed by physics and thermodynamics. A

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survey of the literature over the past 10 years yields many different kinds of devices called concentrators [1–19]. Some of these systems are no more complicated than the simple hand lens. Others, such as the Fluorescent Planar Concentrator, seem, at first glance, to have radically different thermodynamic limitations [6]. All of these systems, however different, are related in that they increase the number of photons on a surface or the irradiance (illumination) above the level present without the device. This is an advantage for solar energy conversion and material characterization, since the end receiver (for example a solar cell or absorber) can be reduced in size relative to the total system. In this way, area exposed to the sun can be covered by potentially cheaper and technologically simple materials. High photon (flux) levels can also be used for the generation of high temperatures for the production of steam, photothermal reactions or materials processing [20]. In this paper will be presented a unification of the principles of geometrical optics for fluorescent concentrators. The general equations developed will be discussed with regards to use in solar energy and solar concentrators.

A concentrator that uses geometry alone to concentrate, and that does not involve a frequency shift will be called a geometrical or passive concentrator. A system that concentrates due to a frequency shift alone will be called a fluorescent, luminescent, or active system. A combined system, with both processes operating in tandem, will be called a hybrid system.

## 2. Geometrical optics

### 2.1. General theory and sine brightness law

Considerations of the flux transfer in a geometrical system leads to an understanding of how a passive concentrator functions. In this paper, subscript 1 refers to the entrance aperture, while 2 shall refer to the exit or absorber aperture. Considering a typical optical system (transformer) with entrance aperture  $A_1$  and exit aperture  $A_2$ , light enters the system within a cone defined by  $\pm\theta_1$  and is collected within  $\pm\theta_2$  as measured from the optical axis (see fig. 1). The brightness, stellar or radiance of the light,  $L$ , is the flux per unit solid angle,  $\Omega$ , per unit projected area [1,20,21,11]. The flux incident on the top aperture from a Lambertian source such as the sun is then given by the integral of the radiance times the area and projected solid angle, or

$$\Phi_1 = \int L_1 A_1 \cos \theta \, d\Omega = \int_0^{\theta_1} 2\pi L_1 A_1 \sin \theta \cos \theta \, d\theta = \pi L_1 A_1 \sin^2 \theta_1.$$

A similar expression is obtained for the exit aperture with subscripts of 2. The concentration ratio,  $C$ , is given by the ratio of the illumination on the exit aperture to the entrance apertures, or

$$C \equiv \frac{\Phi_2/A_2}{\Phi_1/A_1} = \frac{L_2 \sin^2 \theta_2}{L_1 \sin^2 \theta_1}.$$

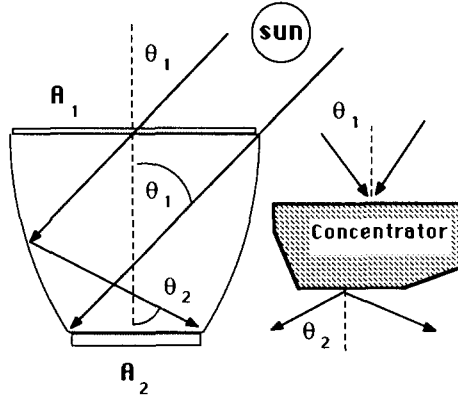


Fig. 1. Typical arrangement for a geometrical or passive concentrator. Light incident on the entrance aperture area  $A_1$  within  $\pm\theta_1$  will reach the exit aperture area,  $A_2$ , within  $\pm\theta_2$ . The system can be comprised of a reflective surface as shown, or can be made up of a lens, mirrors or combinations of geometrical shapes.

In a geometrical system, concentration is obtained by conserving flux throughout the system, as well as the stearence of the light ( $L_2 \leq L_1$ ). This means that as the beam area is decreased, the divergence, or angle, is increased to compensate [1,21,22]. It can then be seen that area is exchanged for angle to achieve concentration. The maximum geometric concentration ratio is then given by

$$C \leq \frac{\sin^2\theta_2}{\sin^2\theta_1} = \frac{1}{\sin^2\theta_1}, \quad (3)$$

where the output angle is usually taken as  $90^\circ$ . Eqs. (2) and (3) are the well known sine brightness equation for ideal geometric flux transfer, which can also be derived using the phase space, or étendue of the light [1,7]. If the concentrator is made of a medium of refractive index,  $n$ , and the exit plane is immersed in this medium as well, then it is necessary to modify the concentrator equation. The edge ray  $\theta_1$  will be refracted to  $\theta'_1$  in the concentrator, where  $\sin\theta_1 = n \sin\theta'_1$  from Snell's law. For a concentrator with the exit aperture immersed in the medium, with  $\theta_2$  unchanged or unrefracted, the concentration is characterized by

$$C \leq \frac{\sin^2\theta_2}{\sin^2\theta'_1} = \frac{n^2 \sin^2\theta_2}{\sin^2\theta_1}. \quad (4)$$

For convenience, the concentration is defined by the maximum incident external angle  $\theta_1$ , and the final exit angle  $\theta_2$ . Comparing this to eq. (2), one concludes that  $n^2 L_1 \geq L_2$  for a passive system. This means that upon crossing into a medium of higher index of refraction, the radiation is confined to a smaller solid angle, and thus will have a higher radiance [21,22]. The ramifications of this will be discussed later. The equations in this paper are presented for a 3D or circularly symmetric concentrator. For a 2D system, where light is reduced in one direction, the concentration is the square root of the 3D value.

## 2.2. Examples of ideal geometric concentrators

Most imaging systems, such as Fresnel lenses and parabolic reflectors, fall short of the limit of eq. (4) by a factor of 4 or more [1,3]. This is partially due to the fact that an image of the sun is transferred to the exit, as well as the flux, and that the exit angle is less than  $90^\circ$ . One type of concentrator that can approach the limit (equality) is the CPC or compound parabolic concentrator, or non-imaging concentrator [1,3–5]. The CPC is a concentrator with a cross section of two parabolic sections each tilted at  $\theta_1$ , such that the focus of each section is in the bottom corner of the other. Light incident on the entrance aperture reaches the exit plane after 1–2 bounces on the reflective side walls (see fig. 1). This concentrator resembles a cone (or trough in 2D). In fact, an approximation of the CPC is often taken as a straight wall cone. As an illustration of the CPC, and of the use of eq. (4), consider a concentrator with  $\theta_2 = 90^\circ$ , as is often the case, and  $\theta_1 = 42^\circ$ . For the moment, let the refractive index,  $n$ , be unity (air). From eq. (4), the concentration limit is 2.2 suns or 2.2 times the incident. If placed in a fixed orientation, the concentrator could collect approximately 6 h during the day, with only minimal seasonal adjustment. Now, if this concentrator is filled with oil of refractive index 1.5, then a ray incident at  $90^\circ$  on the entrance aperture will be refracted to  $42^\circ$ . The concentrator now functions with  $\theta_1 = 90^\circ$ , but the concentration ratio is unchanged at 2.2, which is  $n^2$ . This is because the initial choice of the input angle,  $42^\circ$ , was the critical angle,  $\theta_c = \sin^{-1}(1/n)$ . The concentration is over  $2\pi$  solid angle and the system will accept fully diffuse (cloudy day) radiation [11].

Other systems have been devised that operate in this  $n^2$  limit for concentration. One system is a hemispherical lens of diameter  $nW$ , where  $W$  is the width of the attached absorber or solar cell, and  $n$  is the refractive index of the dome (see fig. 2). Light enters the dome and is refracted to the exit plane. This concentrator has a variable entrance aperture in that different portions are used at different incident angles. For any input angle, the rays that intercept the dome that would otherwise have crossed the diameter area of the hemisphere will reach the immersed exit aperture. This system is one of the few imaging systems that operates close to ideal concentration limits. In fact, if one views the solar cell through the dome at any

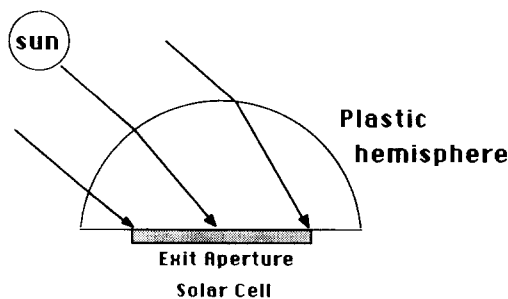


Fig. 2. Hemisphere lens used to produce a concentration ratio of  $n^2$ . Light incident on the dome that would otherwise have hit the diameter or mid-section area will reach the exit aperture.

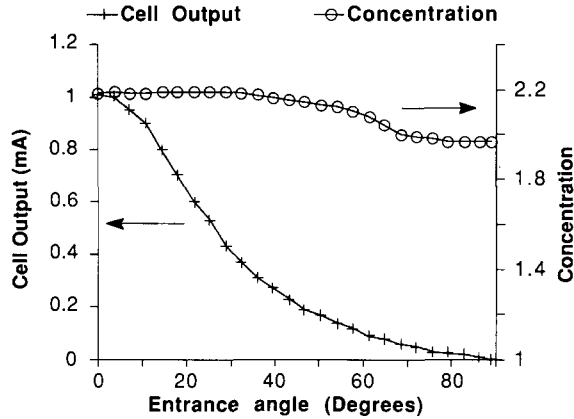


Fig. 3. Experimental data for the hemisphere lens with a 4 mm Si solar cell bonded with epoxy to a 6 mm diameter acrylic dome. The photocurrent is recorded as a function of angle of incidence and referenced to a cell without the device to obtain concentration [11].

angle, one sees a  $2\times$  magnified image. The hemisphere lens has no spherical aberration or coma, and is related to the aplanatic lens used in microscopy [1,22]. Shown in fig. 3 is data obtained for an actual photovoltaic device in this configuration. The concentration is essentially a constant, with respect to entrance angle, near 2–2.2 suns. Note that although the concentration ratio is essentially constant, the output of the device falls off dramatically with the incidence angle. The light source used for the experiment was a Hewlett Packard HLMP-8150 high intensity LED, with essentially all exit radiation collimated in a cone of  $3^\circ$ . For this light emitting diode, the flux varies with  $\cos^n\theta$  ( $n > 1$ ), so that the output of the concentrator and control cell will also follow this variation (compare with eq. (1)). If sunlight is used for the experiment, the output varies with the cosine of the angle, but the concentration ratio is unchanged. This is an important and general conclusion in concentrator technology. A concentrator will only concentrate radiation present at its entrance aperture. Since the illumination at the entrance varies with the entrance angle, so too will the output. In practice, as the angle increases, the transmission losses must also be included [23].

Another example of an  $n^2$  concentrator is a transparent plate with white paint, or a scattering surface, on one side (see left-hand side of fig. 4). Light refracts into the plate and scatters off the bottom surface. A fraction  $\sin^2\theta_c$  of this light escapes, but some light undergoes total internal reflection to be collected at a solar cell bonded to the plate in an unpainted area [11]. This system is non-ideal, but can achieve practical concentrations of 1.7 suns, and is used by some photovoltaic panel manufacturers.

Ideal geometric concentrators have been built that function in the high concentration range (10–100 000 suns) [1,3–5]. These systems are related to the simple CPC, but use reflective hyperbolic or straight side walls, and a lens at the entrance aperture in order to reduce the total system length. High concentration ratio systems

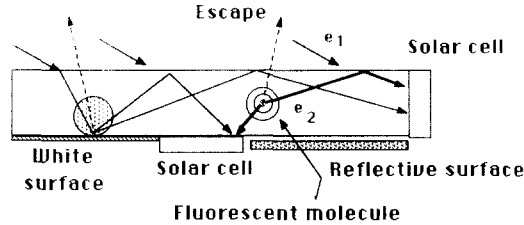


Fig. 4. Operation of a white painted transparent plate  $n^2$  concentrator showing the process of light scattering, escape, and total internal reflection (left-hand side), and active fluorescent planar concentrator, showing the process of photon absorption, emission, and escape (right-hand side). The solar cells represent the possible placement of the exit aperture.

require solar tracking, since the acceptance angle is small, but have achieved 84 000 suns and  $7.2 \text{ kW/cm}^2$  irradiance in experimental facilities. In the high concentration regime,  $\theta_1$  is limited by the suns angular size ( $0.27^\circ$ – $0.26^\circ$ ). For  $n = 1.5$ , this means that concentrations of 100 000 suns are theoretically possible from eq. (4). These high flux levels could be used for a solar pumped laser or in the destruction of hazardous chemical wastes. It is also worth noting that since the non-imaging concentrators operate near the equality in eq. (4), they can also be used in reverse to control the angular output range of a light source. This property makes them interesting for use in collimating light sources such as light emitting diodes [1,5].

### 3. Active concentrators (luminescent systems)

#### 3.1. Theory of luminescent systems

It would seem from the previous section that the limit for diffuse radiation concentration is  $n^2$  [1,11]. This is surpassed in the case of the fluorescent concentrator, which concentrates diffuse or direct radiation to levels above this [13,14]. In the current technique, a plastic or dielectric material is doped with an organic dye or fluorescent inorganic molecule (see right side of fig. 4). Light is absorbed at one energy, and is Stokes-shifted or re-emitted to a lower energy (from the blue to the red). A portion of this light is trapped in the plate via total internal reflection, and can be collected by the absorber plane or exit aperture. In this way, the system resembles the white painted transparent plate described above. Both the entrance and exit angles are  $90^\circ$ . Another similarity is that since the absorption of the incident light is not dependent on the angle, the system can operate at all incidence angles. What is different is that the process is physical or quantum, and does not rely on geometrical optics. This means that the concentration is not limited by eq. (4). In the section that follows, the radiance,  $L$ , will be considered in a small spectral band of energy  $de$ , where  $e$  is the photon energy,  $h\nu$ . The process, which is really a chemical reaction, occurring via the fluorescent molecule is



where  $e_1 > e_2$ . The system operates like an optical heat pump. The radiance at one energy is increased by changing some of the incoming energy to heat.

Consider the process of emission of radiation from a black body radiator such as the sun [21]. The equation governing this is the familiar Planck formula from quantum mechanics:

$$L_x = \frac{2}{h^3 c^2} \frac{e_x^3}{\exp(e_x/kT) - 1}, \quad (6)$$

where  $e$  is the photon energy and  $T$  is the temperature of the solar surface (5777 K). The constants  $h$ ,  $c$ ,  $k$  are Planck's constant, the speed of light in vacuum, and Boltzman's constant, respectively. This equation can be solved in terms of  $T$  for any given spectral radiance,  $L_x$ , to obtain the effective or radiance temperature [7]. The emission of a photon of energy  $e$  from a luminescent molecule can always be assigned some temperature to describe the radiance  $L_x = L_2$ , but it is more useful to describe the process of luminescent emission in terms of the system or ambient temperature  $T_0$ . In this case, a constant  $\mu$  is inserted into the equation to describe the luminescent radiance. This yields for the emission

$$L_x = \frac{2}{h^3 c^2} \frac{e_x^3}{\exp[(e_x - \mu_x)/kT_0] - 1}. \quad (7)$$

Comparing eq. (6) with eq. (7), one finds that  $(e - \mu)/T_0 = e/T$ . A portion  $\mu$  of the original energy,  $e$ , is converted to the excited state of the molecule at temperature  $T_0$ . Since  $e - \mu$  is the heat dissipated in the fluorescent reaction, this equation can be understood as the entropy balance between adsorption or emission. Formally,  $\mu$  is called the chemical potential, the free energy at constant volume, or the ability of the photons to do work at ambient temperature. Eq. (7) is also understood as the distribution of energy one must expect for a Boson gas such as a photon or helium 4 [9,6]. As an aside, one can also include in eq. (7) an emissivity-type term to describe the actual magnitude of the radiance [9,21].

Eq. (7) can be solved in terms of chemical potential to obtain

$$\mu_x = e_x - kT_0 \ln \left( \frac{2}{h^3 c^2} \frac{e_x^3}{L_x} + 1 \right). \quad (8)$$

It can be seen that the chemical potential depends on the photon energy,  $e$ , but also the radiance of the light. Using these concepts, it has recently been shown that the chemical potential of light at the bandgap energy of an ideal solar cell is equal to the maximum open circuit voltage (i.e. 0.82 V for  $E_g = 1.1$  eV) [9]. There is no reason why this equation cannot also be used to describe the incident solar radiation at the earth's surface at ambient temperature. In this case, the chemical potential is analogous to temperature, in that it reflects the ability of one source of light to exchange radiation with another source. In a purely thermal system, radiation exchange cannot taken place between bodies of equal temperature. A chemical system is characterized by the number of photons or chemical species, not the

temperature. It is the chemical potentials in this case which then must be equal at equilibrium. This implies

$$\mu_1 = \mu_2$$

or

$$\left( \frac{2}{h^3 c^2} \frac{e_1^3}{L_1} + 1 \right) / \left( \frac{2}{h^3 c^2} \frac{e_2^3}{L_2} + 1 \right) = \exp \frac{e_1 - e_2}{kT_0}. \quad (9)$$

This equation can be defined as

$$\frac{(P_1 + 1)}{(P_2 + 1)} = H \quad (10)$$

for simplicity. From this equation, the concentration ratio can be obtained as

$$\begin{aligned} C &\equiv \frac{L_2}{L_1} = \frac{P_1}{P_2} \frac{e_2^3}{e_1^3} = \frac{e_2^3}{e_1^3} \frac{HP_1}{1 - H + P_1} \\ &= \frac{e_2^3}{e_1^3} \frac{2e_1^3}{h^3 c^2 L_1} \exp \frac{e_1 - e_2}{kT_0} \left( 1 - \exp \frac{e_1 - e_2}{kT_0} + \frac{2e_1^3}{h^3 c^2 L_1} \right)^{-1}. \end{aligned} \quad (11)$$

This equation was presented by an earlier publication in terms of radiance temperature using a derivation that involves the entropy of the light [7]. Under typical conditions for present luminescent dyes, and incident illumination below 100 suns, this equation can be approximated by

$$C \approx \frac{e_2^3}{e_1^3} \exp \frac{e_1 - e_2}{kT_0}. \quad (12)$$

This equation was previously derived by other means, with the exception that the concentration was defined as the ratio of the number of photons, and not in terms of power or energy as is done in the present publication [6].

### 3.2. Concentration values in fluorescent systems

It is instructive to determine the maximum concentration ratio allowed for a typical luminescent dye in a FPC. Such a dye is Lumogen F Red 300 produced by BASF [24]. This new class of perylene dyes is worthy of future study, since they have recently demonstrated excellent stability in outdoor and UV exposure tests. This result should renew interest in fluorescent concentrators, which have been virtually abandoned due to problems with the bleaching and fading of the dyes. This dye has an absorption maximum near 578 nm ( $e_1 \cong 2.14$  eV) and an emission maximum near 613 nm ( $e_2 \cong 2.02$  eV). To obtain the concentration ratio, one must know the radiance of the incident light in the proper units. The typical AM 1.5 value at this energy is reported as approximately 1200–1300 W/m<sup>2</sup> · μm. [20,25]. This illumination value can be converted to radiance by realizing that it represents the power integrated over all angles. Since illumination,  $E$ , is the power or flux per



unit area, setting  $\theta_1 = 90$  in eq. (1) yields

$$L_\lambda = E_\lambda / \pi. \quad (13)$$

Next, one must convert the radiance from the partial with respect to wavelength to the partial with respect to energy, or

$$L_x = \frac{dL}{de} = \frac{\lambda}{e} \frac{dL}{d\lambda} = \frac{\lambda}{e} L_\lambda. \quad (14)$$

Using 2.14 eV (0.58  $\mu\text{m}$ ), and  $kT_0 = 0.026$  eV, one obtains  $1.07 \times 10^2 \text{ W/m}^2 \cdot \text{eV}$  or  $6.69 \times 10^{20} \text{ eV/m}^2 \cdot \text{s} \cdot \text{eV}$ . The conversion from W to eV/s ( $1.602 \times 10^{-19} \text{ J/eV}$ ) is needed so that  $P_1$  is dimensionless. From this value, one obtains  $P_1$  as  $4.4 \times 10^6$ . As an aside, this value of  $P_1$  corresponds to an equivalent temperature of approximately 1594 K from eq. (6) or a chemical potential of 1.7 eV from eq. (8). One then finds, from eq. (11) or (12), that the concentration is limited to values below 102 suns.

In practice, the measured concentration ratio (2–10 suns) using a solar cell is much less than the value obtained above [13,14]. This is due to several reasons. One must examine the processes occurring in fig. 4 to understand the operation of the practical concentrator. A photon of energy  $e_1$  is absorbed in the concentrator and another is emitted at  $e_2$ . There is a chance that all of the energy will be dissipated as heat, and that not all photons undergo the transformation represented by eq. (5). All dyes have absorption and emission bands that overlap. Thus, some of the emitted light is re-absorbed during the transport to the exit aperture. The re-radiation is isotropic, and only a portion is geometrically confined due to total internal reflection.

Let us examine each of these loss mechanisms. There is an efficiency of the trapping process,  $\eta_{\text{trap}}$  due to the light lost from the plate from the isotropic nature of the emission (75% or  $\cos \theta_c$ ). Each time a photon is absorbed and re-emitted, a fraction  $(1 - \cos \theta_c)$  is lost. The ratio of the photons emitted to the photons absorbed,  $\eta_{\text{lum}}$  can be less than 100%. For recently developed lumogen dyes this term can be 90%, and can be enhanced with the proper chemical design of the dyes. This means that each time light is absorbed 10% of the photons will be dissipated. This loss results in the most serious limitation to the efficiency due to parasitic self absorption of the emitted light or  $\eta_{\text{par}}$  (30–40%). This efficiency can be obtained by raising the product ( $\eta_{\text{lum}}\eta_{\text{trap}}$ ) to the power of the number of absorption encounters of the light with fluorescent molecules in the system. As pointed out in a previous paper, the process of absorption thermodynamically implies that some of the emission is re-absorbed [7,6]. In fact, eqs. (9) and (11) are viewed as the ratio between the Stokes and anti-Stokes reactions implied by the bi-directional arrow in eq. (5). One way to limit the effects of self-absorption is to find materials with high Stokes shifts ( $e_1 - e_2$ ). One can also imagine that an alternative system could be built that does not rely on total internal reflection to trap the light. Such a system could have a large area short wavelength pass selective filter at the entrance aperture that will pass  $e > e_1$  and will reflect  $e < e_1$ . This filter could pass the shortest wavelength

light that could be absorbed by the dye, but reflect the light emitted ( $e < e_2$ ). The value of  $e_2$  could be chosen in a region with minimal overlap of absorption and emission by the selection of the filters. For example, this value could be near the edge of the emission spectrum. Instead of being limited by the absorption and emission maximum (characteristic peaks) of the dye, with such a filter mirror system, one could have additional freedom in the choice of  $e_1 - e_2$ . The system would then be limited by the entrance filter reflectivity to the emitted radiation raised to the power of the average number of bounces in the plate or concentrator before collection at the exit aperture. However, for a number of reflections greater than 10, large concentration ratios, the trapping efficiency would again be less than 60% for an entrance filter reflectivity of 90%. Note that the average number of reflections is of the order of the geometrical ratio  $A_1/A_2$ , so that this approach is also limited to low efficiencies unless cheap high reflectivity filters can be found. Other problems with the fluorescent concentrator include the fact that only a portion of the incoming solar spectrum is absorbed by the dye (10%–22%), so the absorption efficiency,  $\eta_{\text{abs}}$ , is low.

Not all effects decrease the performance of the FPC. Since the detector is attached to the refractive plate, an additional  $n^2$  factor is gained from eq. (4). Finally, a small enhancement,  $\epsilon$  (100%–110%), is expected if a solar cell is used to measure the concentration, due to the fact that absorption of the light in the detector can occur at a more favorable depth relative to the depletion region [25,26]. If one uses the example of the lumogen dye (102 suns) for the concentration ratio of a FPC, one obtains for the estimated measured concentration,  $G$ , 4–10 suns from

$$G \approx C_{\text{lum}} n^2 \eta_{\text{trap}} \eta_{\text{abs}} \eta_{\text{lum}} \eta_{\text{par}} \epsilon. \quad (15)$$

The actual experimental concentration in a state of the art FPC is in the range 2–10 suns [13,14]. This value, while low, illustrates the advancements that can be made by further research in active concentrator systems.

#### 4. Discussion

Much of this paper has focused on applications regarding solar cells. However, all of the results presented can be applied to thermal systems as well. As mentioned in section 2.1, the radiance or brightness of light is increased upon crossing into a higher refractive index material. This effect is worth mentioning in the light of recent experimental results that have shown that irradiance or illumination ( $\text{W}/\text{m}^2$ ) levels obtained by concentration can exceed that of the sun's surface [3]. This result supports, rather than contradicts, thermodynamics. The radiance of a blackbody is given by eq. (6). In order to obtain the irradiance at the sun's surface, this equation must be integrated over  $2\pi$  solid angle and energy to obtain the familiar equation  $\sigma T^4$ , where  $\sigma$  is the Stefan–Boltzman constant, and  $T$  is the temperature of the sun's surface [20,21]. We will assume unit emissivity and absorptivity for this discussion. For the irradiance,  $E$ , at the earth's surface, eq. (6) must first be

multiplied by the dilution factor,  $f$ , representing the inverse square law fall off of the solar radiation at the earth–sun distance. The solar size at the earth’s surface ( $\theta_{\text{sun}} = 0.27^\circ - 0.26^\circ$ ) results in a dilution factor of  $1/46\,000$ . This value results in a value  $E$  of the solar constant,  $1350 \text{ W/m}^2$ , just outside the earth’s atmosphere. To obtain the irradiance in a concentrator of refractive index  $n$ , one must realize that the  $c^2$  term in eq. (6) is the group velocity in vacuum, and must be replaced with  $c^2/n^2$  in a dielectric medium. In this convention, eq. (6) is then multiplied by  $fn^2$ , where  $f$  is defined as less than 1 [20]. The maximum irradiance inside the concentrator is then given by the integration of eq. (6) multiplied by  $fn^2$ , where  $f$  is given by  $C \sin^2\theta_{\text{sun}}/n^2$ . The maximum thermodynamic concentration ratio,  $C_{\text{max}}$ , is given from eq. (4), by setting  $\theta_2 = 90^\circ$  and  $\sin^2\theta_1 = \sin^2\theta_{\text{sun}}$ . This results in a value of  $46\,000n^2$  for  $C_{\text{max}}$ . The maximum value of the irradiance is then  $n^2\sigma T^4$ , which is  $n^2$  times the solar surface irradiance. Note that the equilibrium temperature of the absorber,  $T_2$ , using concentration can be found by equating the flux output per unit area at the absorber and solar surface,

$$\sigma T^4 = n^2 f \sigma T_2^4, \quad (16)$$

which shows that the irradiance inside the concentrator cannot produce a temperature higher than the sun’s even if the irradiance is higher. In other words, since a black body will radiate more when immersed in a refractive medium, solar absorber temperatures can never exceed that of the sun (5777 K).

It is worth noting that the radiance temperature given by eq. (6) can be quite high for luminescent systems. In fact this effect has been used to obtain temperatures of  $300\text{--}500^\circ\text{C}$  [13]. Of course, one can also easily obtain this by use of a small geometric concentrator alone and spectrally selective absorber. The concept of equivalent radiance temperature can be used as follows [7,20]: From Kirchoff’s law recall that the emissivity and absorptivity are equal. From section 3.1 one sees that the equivalent temperature for 2.1 eV photons at AM 1.5 illumination is 1594 K. The temperature of photons of higher energy is larger for the solar spectrum. This, if one uses a selective (and evacuated) absorber with an absorptivity of 1 above 2.1 eV, and a reflectivity of 1 for photons below this energy, equilibrium, temperatures of 1600 K are possible in principle even with low concentration [7].

One must also make a distinction between concentration, which has been treated in this paper as a thermodynamic property, and enhancement, which compares the amount of absorption compared to some reference. For example, if one uses a concentrator and a bifacial solar cell or two-sided exit aperture, the concentration ratio is given by the equations in this paper. However, the enhancement can be given by as much as 2 times the concentration ratio, since the surface area is doubled. Likewise, for volume absorption in textured sheets, it has been shown that the enhancement can be 4 times that expected from concentration alone,  $n^2$  [15–18]. This is a subtle point, but one finds that it is better to separate the concepts of concentration, which references to the unidirectional solar radiation, and enhancement, which can be dependent on specific geometry.

## 5. Conclusions

Concentration ratios,  $C$ , have been established for passive and active systems. The concentration ratio has been shown to depend on the angular collection range, the index of refraction and the frequency or Stokes shifts in the concentrator. As an example of a hybrid or combined system, one can consider the case of a solar powered laser [27,28]. Using the optics described in this paper, it should be possible to describe the laser operation in terms of very high concentration from the product of passive and active system concentration. (eq. (4) times eq. (11)). In other words, the condition for laser operation is that the chemical potential of the incident radiation must be equal to the energy of the emitted (laser) photon. Such a system is shown in fig. 5 for a combination of ideal high geometric ratio system with luminescent concentrator. When the proper condition is met, the output of the fluorescent concentrator would appear as collimated light from the edge of the system. Hybrid systems could extend the range of useful materials for solar lasers beyond that previously described [19]. Practical ideal concentrators have been built for geometrical concentration alone. These systems resemble, in some ways natural systems [1,29]. There is, however, much room for improvement in luminescent systems, which now operate far from ideality. Future work in the field of solar energy materials could concentrate on this to obtain a brighter way to collect light.

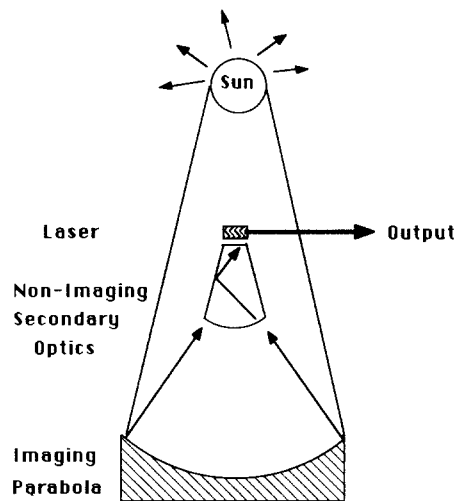


Fig. 5. Arrangement of a near ideal hybrid passive-active system to obtain solar laser light. Light is focused on the non-imaging concentrator by the parabola, where it can be directed to a fluorescent (laser) material [1,3,4].

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