Accelerating Reference Frame for Electromagnetic Waves in a Rapidly Growing Plasma: Unruh-Davies-Fulling-DeWitt Radiation and the Nonadiabatic Casimir Effect

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Shortly after Hawking's prediction of thermal radiation from black holes, it became apparent that observers in *accelerating frames* should see a Planck distribution of electromagnetic radiation (Unruh radiation). Since an acceleration $g = 980$ cm/sec^2 produces a radiation temperature of only $\sim 4\times10^{-20}$ K, the detection of such thermal radiation is a major challenge. A nonlinear optical medium whose index of refraction is changing rapidly with time accelerates zero-point quantum fluctuations. The sudden ionization of a gas or a semiconductor crystal to generate plasma on a subpicosecond time scale can produce a reference frame accelerating at $\sim 10^{20}g$ relative to an inertial frame.

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In 1974, Hawking showed[1,2] that black holes can evaporate by the emission of low-temperature thermal radiation, now named Hawking radiation. Shortly thereafter, a closely related effect called Unruh radiation became apparent. According to Unruh[3] and Davies,[4] observers of the electomagnetic field in an *accelerating reference frame* would see thermal radiation at a temperature T,

$$kT = (\hbar/2\pi)(a/c)\,, \tag{1}$$

where a is the acceleration relative to an inertial frame, c is the speed of light, and $\hbar$ and k are Planck's and Boltzmann's constants, respectively. In a frame accelerating at $g = 980$ cm/sec^2, equivalent to the acceleration experienced at the Earth's surface,[5] this thermal radiation is at a temperature of only 4×10^{-20} K. Therefore, physicists hoping to observe this radiation have sought out systems being subjected to extreme acceleration. For example, Bell and Leinass have suggested[6] that the spin depolarization of electrons accelerating around a synchrotron storage ring may be interpreted as being due to such radiation.

The basic physical mechanism for this radiation is that zero-point quantum fluctuations in an accelerating frame become transformed into real thermal photons when Fourier analyzed in terms of the photon modes (plane-wave basis set) of an internal frame. The observer need not experience the accelerating directly. DeWitt[7] and Davies[8] argued that the zero-point field fluctuations near an accelerating mirror are already being subjected to an accelerating motion. In effect, the moving mirror forces a relative accelerating motion between the electromagnetic quantum fluctuations and a stationary observer. Remarkably, the conversion of zero-point quantum fluctuations into real thermal photons is equally effective whether the mirror is advancing toward the observer or receding away from him.

Of course, the acceleration of a physical mirror is subject to the same pragmatic limitations as the acceleration of a physical observer. In this regard, we believe that modern nonlinear optical techniques can help make Unruh-Davies-Fulling-DeWitt (UDFD) radiation experimentally accessible. Ideally, in nonlinear optics, we regard the refractive index of a medium to be a time-variable function totally under the control of the experimentalist. If we observed the zero-point electromagnetic field transmitted through a window whose index of refraction is falling with time, the phase shift of the field is the same as when reflected from an accelerating mirror. In this sense, nonlinear optics is the experimentalist's way to produce fast-moving mirrors.

Most nonlinear optical effects tend to be small and oscillatory. There is, however, at least one nonlinear optical effect which is very large and monotonic. When a gas is suddenly photoionized, it turns into a plasma and its index of refraction drops from 1 to 0. Some time ago, the phase modulation associated with such a laser breakdown plasma in a gas was shown[9] to produce nearly a 10% blue shift in the transmitted laser photon energy. This is a very substantial effect. Likewise, in a semiconductor, the sudden creation of electron-hole pairs can reduce the refractive index from ~ 3.5 to 0 in a very brief time period. When the electron-hole plasma is induced by subpicosecond optical pulses, the phase modulation can suddenly sweep up low-frequency waves by many octaves. Indeed by lateral synchronization of the excitation process, the moving plasma front can act as a moving mirror exceeding the speed of light. Therefore we can regard such a gas or semiconductor slab as an observational window on accelerating electromagnetic fields.

The relative motion of the observer and the field is controlled by the rate of refractive index change or alternatively by the time-varying plasma density. In this paper we will (i) attempt to solve for the time evolution of the quantum field operators in a time-varying index of refraction, and (ii) determine the feasibility of detecting the UDFD radiation in a low-temperature semiconductor experiment. We hope to show that an acceleration of $\sim 10^{23}$ cm/sec^2 is feasible, and that the corresponding UDFD radiation intensity should be sufficient to allow experimental observation.

Let us consider a simple phase-modulation model for plane waves in a time-varying index of refraction $n(t)$. The electromagnetic field at an angular frequency ω and wave vector $k=n_0\omega/c$ is transmitted through a slab of material of thickness z with initial refractive index n_0. The decrease of refractive index with time t is expressed as a Taylor series, $n(t)=n_0-\dot{n}t$. The phase factor of the wave becomes

$$\exp\left\{i\left[kz-\omega t\right]\right\}=\exp\left\{i\left[\frac{n_0\omega}{c}z-\frac{\dot{n}\omega z}{c}t-\omega t\right]\right\}. \quad (2)$$

The frequency emerging from the slab of material is shifted up according to $\omega\to\omega(1+\dot{n}z/c)$, which resembles a Doppler shift for a velocity $\dot{n}z$. Now allow the thickness of the material slab z to become a variable quantity which tracks a phase front $z=ct/n$ of the electromagnetic wave. Then the instantaneous frequency of a phase front passing through the material is $\omega\to\omega[1+(\dot{n}/n)t]$, which resembles the Doppler-shifted frequency $\omega[1+(a/c)t]$ seen by an accelerating observer. Because of the frequency sweep, these are known as a "chirped" waves in nonlinear optics. Comparing these frequencies, we see that a rapidly developing "plasma window" induces a relative acceleration between an observer and the electromagnetic field of $a=c(\dot{n}/n)=c(\dot{\omega}/\omega)\approx c/\tau$. If the characteristic time τ is in the subpicosecond range, then an acceleration of $\sim 10^{20}g$ is feasible. Likewise, a more complicated frequency chirp which tracks a relativistic acceleration can be accommodated by including more terms in the Taylor series expansion for $n(t)$.

To consider the field evolution in a plasma more rigorously, let us write Ampere's law in the transverse Coulomb gauge for the vector potential $\mathbf{A}$, $B=\nabla\times\mathbf{A}$ and $\nabla\cdot\mathbf{A}=0$:

$$\nabla\times(\nabla\times\mathbf{A})+(1/c^2)\partial^2\mathbf{A}/\partial t^2-(4\pi/c)\mathbf{J}=0. \quad (3)$$

The effects of the plasma are represented by the current $\mathbf{J}=Ne\mathbf{V}$, where $N(t)$ is the plasma density and $\mathbf{V}$ is the fluid velocity. Using $m\dot{\mathbf{V}}=e\mathbf{E}=(-e/c)\dot{\mathbf{A}}$, $\mathbf{V}$ can be written as $(-e/mc)\mathbf{A}$ and eliminated from Eq. (3). The time variation $N(t)$ expresses itself mainly through the transverse electromagnetic wave frequency dispersion relation $\omega^2(t)=\omega_p^2(t)+k^2c^2$, where the plasma frequency $\omega_p^2=4\pi Ne^2/m$, and e and m are the electronic charge and mass, respectively. Alternatively, the dispersion relation could be derived from a dielectric constant: $\omega^2(t)=k^2c^2/n^2(t)$. In the discussion which follows, we will assume that the refractive index $n(t)$ could go both up and down at the will of the experimentalist.

If a complete set of classical solutions $\bar{\mathbf{A}}_k(z,t)$ of Eq. (3) can be found, then field quantization[10] is straightforward: A quantum field operator $A(z,t)$ is introduced, which obeys Maxwell's equations. It makes sense to expand $A(z,t)$ in terms of the $\bar{\mathbf{A}}_k(z,t)$ which are solutions of Eq. (3),

$$A(z,t)=\sum_{\text{all }k}[b_k\bar{\mathbf{A}}_k(z,t)+b_k^\dagger\bar{\mathbf{A}}_k^*(z,t)], \quad (4)$$

where b_k and $b_k^\dagger$ are the annihilation and creation operators, respectively, appropriate to this basis set. Alternatively, the field operator may be expanded in terms of a conventional plane-wave basis set. With the usual choice of normalization,

$$\mathbf{A}_k(z,t)=c[2\pi(\hbar/\omega)]^{1/2}\exp[i(kz-\omega t)], \quad (5)$$

$$A(z,t)=\sum_{\text{all }k}[a_k\mathbf{A}_k(z,t)+a_k^\dagger\mathbf{A}_k^*(z,t)], \quad (6)$$

where the a_k and $a_k^\dagger$ are the annihilation and creation operators, respectively, appropriate to the unperturbed vacuum. A comparison of Eq. (4) with Eq. (6) shows that the conventional plane waves $\mathbf{A}_k(z,t)$ evolve into $\bar{\mathbf{A}}_k(z,t)$ waves as the index of refraction changes. Generally the b_k will differ from the a_k, but they are related by a Bogolyubov transformation

$$b_k=\sum_{k'}(\alpha_{kk'}^*a_{k'}-\beta_{kk'}^*a_{k'}^\dagger). \quad (7)$$

A key role is played by the coefficients $\beta_{kk'}$ which measure the admixture of annihilation and creation operators, or equivalently the admixture of positive and negative frequency modes caused by the perturbed vacuum. The expectation number of the accelerated mode k as expressed in terms of the conventional vacuum state $|0\rangle$ is

$$\langle 0|b_k^\dagger b_k|0\rangle=\sum_{k'}|\beta_{kk'}|^2. \quad (8)$$

For us the challenge is to calculate the classical fields $\bar{\mathbf{A}}_k(z,t)$ and then the Bogolyubov coefficients $b_{kk'}$. In the case of an instantaneous change in plasma density, a classical solution has recently[11] appeared. Likewise, a step-function index change can be solved classically. A simple Bogolyubov transformation of that classical solution yields an admixture of annihilation and creation operators: $\alpha_{kk'}=\delta_{kk'}(n+n_0)/2(n_0n)^{1/2}$ and $\beta_{kk'}=\delta_{kk'}(n-n_0)/2(n_0n)^{1/2}$, where $n-n_0$ is the instantaneous step change of refractive index and $\delta_{kk'}$ is the Kronecker δ. In this, our simplest and most important case, we see that the induced photon occupation number will be of order unity.

An interesting special case is the opposite limit: slowly varying plasma density. Then it makes sense to write the time-dependent solution of Eq. (3), $\bar{\mathbf{A}}_k(z,t)$, as a slowly varying amplitude times a rapidly varying phase,

$$\bar{\mathbf{A}}_k(z,t)=A(t)\exp\left\{i\left[kz-\int_0^t\omega(t')dt'\right]\right\}. \quad (9)$$

Substituting Eq. (9) into Eq. (3) we find an equation for the slowly varying amplitude $A(t)$,

$$\ddot{A}-2i\omega\dot{A}-i\dot{\omega}A=0. \quad (10)$$

In the limit of moderate acceleration we may drop the $\ddot{A}$

term in Eq. (10), as is usual in the slowly varying amplitude approximation. Equation (10) then reduces to $2\omega\dot{A} = -\dot{\omega}A$ or $A(t) \propto [\omega(t)]^{-1/2}$. $\overline{\mathbf{A}}_k(z,t)$ can be written

$$\overline{\mathbf{A}}_k(z,t) = c[2\pi\hbar/\omega(t)]^{1/2}\exp\left\{i\left[kz - \int_0^t \omega(t')dt'\right]\right\}. \tag{11}$$

This limit, as represented by Eqs. (9)–(11), mathematically resembles the accelerating mirror[8] and black hole[2] problems which yield a Bose spectrum. This would not be expected in real, finite-duration experiments, since an *exact* thermal spectrum evolves only asymptotically at long times.

Generally, we would have to propagate the field $\overline{\mathbf{A}}_k(z,t)$ out of plasma, through a boundary, and into a conventional vaccum, or deal with a more complex geometry and/or time dependence. In principle, the field operators would follow the same time evolution as the classical fields and the Bogolyubov coefficients $\beta_{kk'}$ would be calculable. The presence of boundaries in the experiments can make the Bogolyubov transformation rather difficult.

Since a plasma expels electromagnetic radiation below its plasma frequency, it modifies that configuration of zero-point quantum fluctuations responsible for the Casimir[12] forces. If the changes are slow enough, the response of the vacuum fluctuations is adiabatic. On the other hand, we are considering sudden nonadiabatic changes which have the effect of causing real transitions and boosting the quantum fluctuations into real photons. In that sense this process may be called the dynamic or nonadiabatic Casimir effect.

We will now consider the experimental parameters required to make these effects observable. The very weak excitation and long wavelength of UDFD radiation seem most consistent with a semiconductor plasma in the presence of a low-radiation background and lattice temperature ~ 1 K. A key requirement is that the UDFD radiation not be overwhelmed by bremsstrahlung emission due to electron-hole collisions in the plasma. Bremsstrahlung emission is linked to momentum relaxation and dissipative absorption through Kirkhoff's law or equivalently, the fluctuation-dissipation theorem. In effect the requirement is that the plasma response be purely reactive and nondissipative, similar to the constraints on the nonlinear optical response in other vacuum modification processes such as optical squeezing.[13]

One must distinguish between real electron-hole excitation by a pump laser tuned in the bands and virtual electron-hole excitation by a laser tuned just below the Urbach tail of the band edge. Real electron-hole pairs are produced by dissipative absorption of the pump laser and are subject to momentum relaxation. Some long-wavelength bremsstrahlung emission is inevitable for this case. Virtual electron-hole pairs, on the other hand, do not have this problem since their response is purely reactive.

It is possible to design an experiment which would discriminate against the bremsstrahlung emission produced by real electron-hole pairs. The carrier kinetic energy should be adjusted to present an optimal balance between Coulomb collisions and deformation-potential scattering. The injected carrier density should be no more than the density required to produce a plasma frequency equal to the observation frequency. Since UDFD radiation responds instantly, and momentum relaxation takes time, gating in the time domain would also be essential. Finally, heterodyne detection would discriminate the phase structure of UDFD radiation from the incoherent bremsstrahlung emission. Since the length limitations of this Letter do not permit a full discussion of this approach, we will emphasize the case of virtual electron-hole excitation.

The Urbach absorption edge in a high-quality semiconductor crystal that is cooled to ~ 1 K can be very sharp. By tuning a pump laser into the transparent region just below the Urbach tail of a direct-gap semiconductor, a large population of virtual electron-hole pairs can be induced. The virtual occupation probability per state is $|\zeta E(\omega_p)|^2/|\hbar\Delta\omega|^2$, where ζ is the transition dipole moment, ω_p is the pump frequency, and $\Delta\omega$ is the detuning. Such virtual electrons are highly polarizable since they respond as if they were bound by small detuning energy $\hbar\Delta\omega$. Although their response is opposite to that of free carriers, the rising and falling edges of the

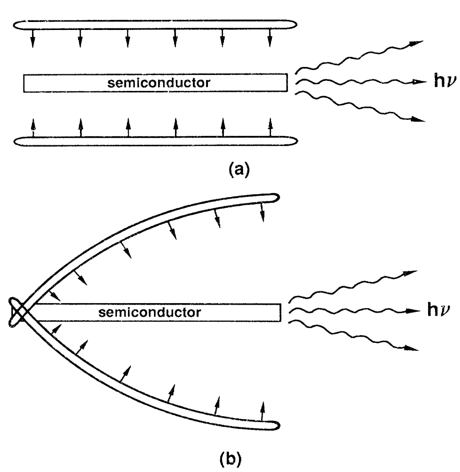

FIG. 1. (a) A geometry for the acceleration of electromagnetic quantum fluctuations along a semiconductor dielectric waveguide due to plasma formation. The direction of the subpicosecond pump pulses is shown by the small arrows. $h\nu$ represents the Unruh radiation. (b) By curving the pump wave, the plasma front can appear to move faster than the speed of light.

pump pulse will supply both signs of acceleration.

This high polarizability can be described in terms of a conventional third-order nonlinear optical susceptibility $\chi^{(3)}(-\omega_u, \omega_u, -\omega_p, \omega_p)$, where ω_u is the very low UDFD frequency. Since the frequencies ω_p, ω_u, and $\omega_p+\omega_u$ are all in the transparent gap of the semiconductor, the response is purely reactive. This $\chi^{(3)}$ can be regarded as a quadratic electro-optic effect similar to the Kerr-type nonlinearity of CS_2. But it can be much larger due to its electronic *doubly resonant* character in ω_p and ω_u. The idea would be to make a near instantaneous step-function change in the refractive index. As shown above, this induces ~ 1 photon per mode, which can be detected by beating against a local oscillator. For example, optical rectification by means of a $\chi^{(2)}$ process in the same crystal can provide a local oscillator against which to beat the UDFD signal. In this respect the experiment would resemble a kind of single-cycle microwave squeezing.[13]

Optical excitation allows interesting flexibility with regard to the geometric and temporal reconfiguration of zero-point quantum fluctuations. Figure 1(a) shows the simplest possible geometry. A thin semiconductor wafer is bombarded on both sides by a planar excitation pulse, the idea being to commence electron-hole excitation simultaneously throughout the sample. This mimics the spatial independence of $N(t)$ in Eq. (3). The far-infrared dielectric waveguide modes of the semiconductor slab would be accelerated and decelerated. Figure 1(b) shows a traveling-wave plasma excitation which is synchronized to exceed the speed of light. Mirrors produce the concave shape of the excitation pulses. This is not a Cherenkov effect since polarizability, not polarization, is moving.

From the above discussion it is clear that there are a number of different ways of looking at this phenomenon: It can be regarded as (1) Unruh-Davies-Fulling-DeWitt radiation, (2) the nonadiabatic Casimir effect, (3) single-cycle microwave squeezing, or (4) the inverse quadratic electro-optic effect with zero-point-photon input waves.

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