Intensity Enhancement in Textured Optical Sheets for Solar Cells

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Abstract—We adopt a statistical mechanical approach toward the optics of textured and inhomogeneous optical sheets. As a general rule, the local light intensity in such a medium will tend to be $2n^2(x)$ times greater than the externally incident light intensity, where $n(x)$ is the local index of refraction in the sheet. This enhancement can contribute toward a $4n^2(x)$ increase in the effective absorption of indirect-gap semiconductors like crystalline silicon.

I. Introduction

In the past decade, there have been a number of suggestions for the use of light trapping by total internal reflection to increase the effective absorption in the indirect-gap semiconductor, crystalline silicon. The original suggestions [1], [2] were motivated by the prospect of increasing the response speed of silicon photodiodes while maintaining high quantum efficiency in the near-infrared.

Subsequently, it was suggested [3] that light trapping would have important benefits for solar cells as well. High efficiency could be maintained while reducing the thickness of semiconductor material required. Additionally, the constraints on the quality of the silicon could be relaxed since the diffusion length of minority carriers could be reduced proportionate to the degree of intensity enhancement. With such important advantages, interest in this approach has continued, but progress in this field has been hindered because there was no method available to calculate the degree of enhancement to be expected.

For example, St. John [1] mentions that total internal reflection will result in two or more passes of the light rays with a proportionate intensity enhancement. On the other hand, Redfield [3] regards the number of light passes, or degree of enhancement, as an adjustable parameter which could vary anywhere between 1 and 100 and he plots the collection efficiency as a function of this parameter. In calculating the ideal efficiency of silicon solar cells, Loferski *et al.* [4] seemed to imply that perfect light trapping might be possible, which corresponds to an infinite degree of enhancement.

It is the purpose of this paper to show that the degree of intensity enhancement to be expected due to total internal reflection is $2n^2$ which for silicon is ≈ 25. In other words, a light ray in silicon may be expected to make 25 passes on average before escaping. With a proper angular average of the longer path length of oblique rays, the "effective" absorption enhancement factor is $4n^2$ over the case of single pass normally incident rays. Also, we will show that the geometrical details of the silicon shape are relatively unimportant. Whether the silicon is simply roughened on one side [1], [2] or whether precisely angled grooves are etched into the surface [3], [4], the overriding tendency will be for an enhancement factor of $4n^2$ provided only that the surface is sufficiently strongly textured. Finally, we will show that in an inhomogeneous sheet, for example a composite, the enhancement will be given by the same formula employing the local index of refraction $n(x)$.

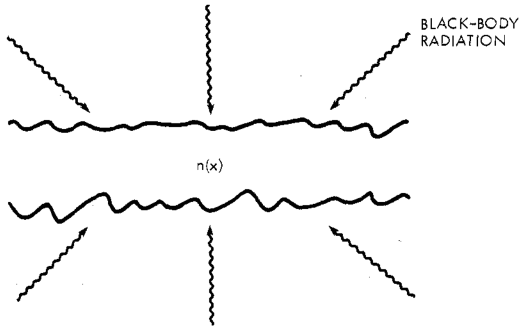

Fig. 1. An inhomogeneous and textured optical sheet with position-dependent index of refraction $n(x)$ immersed in black-body radiation.

Two distinct derivations will be presented in the following two sections. In Section II, we will give a derivation based on statistical mechanics. Such an approach is very powerful and it can be generalized to situations where geometrical optics is inapplicable (though we will not attempt such a generalization in this paper). In Section III, a geometrical optics derivation will be presented. Its simplicity will permit us to better recognize some of the prerequisites and limitations of this type of intensity enhancement. Finally, in Section IV, we will show how these considerations are modified in the presence of absorption. Two specific solar cell structures will be mathematically analyzed.

II. Statistical Mechanical Derivation

Consider an inhomogeneous optical slab with position-dependent index of refraction $n(x)$ as illustrated in Fig. 1. Let the index of refraction vary sufficiently slowly in space, so that a density of electromagnetic modes may be defined, at least locally inside the sheet.

Now place the optical medium into a region of space which is filled with black-body radiation in a frequency band $d\omega$ at a temperature T. When the electromagnetic radiation inside the medium approaches equilibrium with the external black-body radiation, the electromagnetic energy density [5] is

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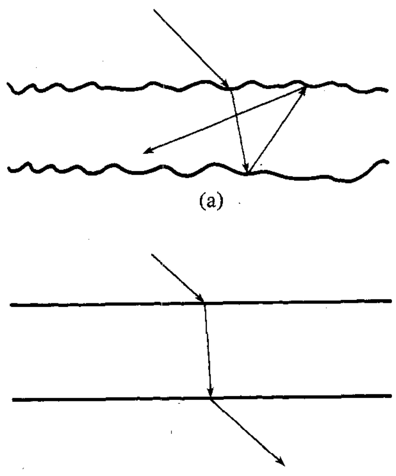

Fig. 2. Two optical sheets with qualitatively different surface textures. (a) Angular randomization and intensity enhancement do occur. (b) In a plane-parallel slab, there is no angular randomization and no intensity enhancement.

$$U = \frac{\hbar\omega}{\exp\{\hbar\omega/kT\} - 1} \frac{d\Omega k^2\, dk}{(2\pi)^3}. \tag{1}$$

This is the standard Planck formula for black-body radiation in a vacuum, but as Landau and Lifshitz show [5], it can be adapted to any optical medium by making $k = n\omega/c$. In addition, the energy density may be changed to an intensity I (power per unit area), by multiplying (1) by the group velocity $v_g = d\omega/dk$. Making both changes in (1) we obtain

$$I \equiv Uv_g = \frac{\hbar\omega}{\exp\{\hbar\omega/kT\} - 1} \frac{d\Omega n^2\, \omega^2}{(2\pi)^3\, c^2}\, d\omega. \tag{2}$$

This differs from the vacuum black-body intensity simply by the factor n^2. Therefore, the intensity of light in a medium which is in equilibrium with external black-body (bb) radiation is n^2 times greater

$$I_{\text{int}}(\omega, x) = n^2(\omega, x)\, I_{\text{ext}}^{\text{bb}}(\omega). \tag{3}$$

This factor comes about simply due to the fact that the density of states in such a medium is proportional to n^2 and the equipartition theorem guarantees equal occupation of the states, internal as well as external.

Now let us decide whether or to what degree the situation changes when an arbitrary external radiation field replaces the black-body radiation. Since we are considering a transparent medium, inelastic events such as absorption and reemission at another frequency are not permitted. Therefore, each spectral component may be considered individually. In this circumstance, departure of the external field from an exact black-body frequency distribution will not affect (3), which will remain valid separately at each frequency ω.

A much more serious question, which will be the main focus of this paper, is: What happens when the external radiation field departs from the isotropic distribution of black-body radiation? This situation is illustrated in Fig. 2(a), where the external light is shown to be collimated. If the surface of the optical sheet is quite irregular in shape, then the light rays, upon entering the medium, will lose all memory of the external incident angle after the first, or at most the second, scattering from a surface. In other words, all correlation with the external angle will be lost almost immediately upon refraction or total internal reflection, especially when averaged over the illuminated surface of the sheet. If this condition is satisfied, then a collimated incident beam of intensity I_{ext} will produce, inside the optical sheet, a random angular distribution of light, no matter which direction the beam happens to be coming from. Therefore, a collimated beam, when subdivided so that it illuminates the optical medium equally from all directions produces identical internal light distributions under those respective conditions. The condition of isotropic illumination is, however, equivalent to that of black-body illumination. Therefore, (3) remains valid whether the external field is isotropic as in the black-body case or whether it is collimated.

$$I_{\text{int}}(\omega, x) = n^2(\omega, x)\, I_{\text{ext}}(\omega). \tag{4}$$

Equation (4) will be corrected for certain surface transmission factors in Section III, but it is a key formula in this paper. It rests on the assumption that all correlation of the internal rays with the external angle of incidence is lost almost immediately upon entering the medium and/or upon averaging over the illuminated surface. Even optical sheets with ordered surface textures will show the type of randomization we are discussing here. The reasons are as follows:

a) If light randomization does not occur upon the entering refraction, it can occur on the first internal reflection.

b) If not on the first reflection, then on the second.

c) If not then, it can still be the result of a spatial average over the illuminated surface area.

d) If not even then, it can still result from angular averaging due to motion of the source, like the sun moving through the sky.

In other words, there is a rather overwhelming tendency toward randomization in the angular distribution of light and toward the validity of (4), but it is not always satisfied.

Consider the simple plane-parallel slab shown in Fig. 2(b). Clearly, there is no intensity enhancement in that case.[1] To distinguish between the class of geometries for which (4) is valid and the class of geometries in which it is invalid is a problem in ergodic theory and in measure theory. We will not attempt in this paper to distinguish between these two classes mathematically in the measure theory sense. Instead, we will assume that the statistical approach is valid except in those few geometries where a cursory inspection shows that randomization cannot occur under any of the circumstances listed earlier, a) to d). Equation (4) will be valid provided any of those prerequisites, a) to d), is satisfied.

Now consider the situation shown in Fig. 3. In that geometry, the light is confined to a half space by the presence of a white reflective plane. In effect, the light intensity external to the optical sheet has been doubled by virtue of reflection from the white surface. The total intensity enhancement will then be given by

[1] Another situation in which (4) is not valid is an optically thick turbid sheet illuminated from only one side. In this case, angular averaging is no problem, but spatial averaging will be incomplete. The side of the sheet away from the source of illumination will be dark.

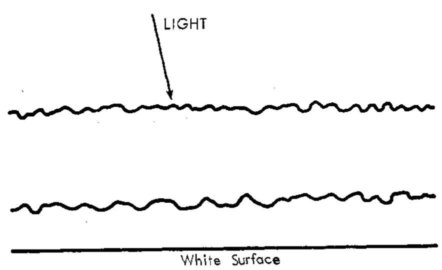

Fig. 3. A white reflective surface effectively doubles the external intensity and increases the enhancement factor to $2n^2$.

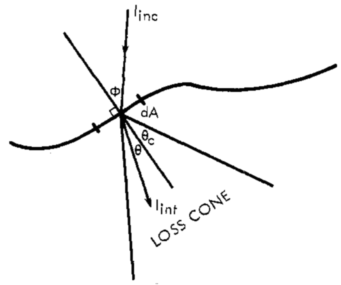

Fig. 4. The balance between incoming and outgoint radiation determines the internal intensity I_{int}.

$$I_{int}(\omega, x) = 2n^2(\omega, x) I_{inc}(\omega) \tag{5}$$

where $I_{inc}(\omega)$ is the incident light intensity.

One of the key implicit assumptions that we have not emphasized earlier is that there be no absorption, either within the volume or on the surface of the medium. Optical absorption effects will be treated in Section IV.

The main conclusion of this section is that a statistical mechanical approach results in the intensity enhancement given in (5). The factor $2n^2$ can be quite substantial; i.e., approximately 25 for silicon and approximately 15 for TiO_2. Even conventional glass with an index of 1.5 has an enhancement factor equal to 4.5. In view of the importance of this result, we give an alternative derivation based on geometrical optics in the next section.

III. Geometrical Optics Derivation

Our approach is based on "Detailed Balancing" of the light which is incident on a small area element dA, and the light in the loss cone which escapes from it. Consider the geometry shown in Fig. 4. Let I_{inc} be the incident radiation power per area element dA. A fraction $T_{inc}(\phi)$ of this light will be transmitted through the incoming interface, where ϕ is the angle of incidence. This must be balanced by the internal radiation which escapes. Let us assume that the internal radiation is isotropic due to the randomizing influence of refraction and reflection from the textured interfaces as discussed in the previous section. Let B_{int} be the internal intensity per unit internal solid angle. The internal intensity I_{int} on both sides of an area element dA is given by

$$I_{int} = \int B_{int} \cos\theta \, d\Omega$$

where $\cos\theta$ is the reduction of intensity on the area element due to oblique incidence. In this paper we will follow the convection that internal intensity I_{int} is bidirectional while the incident intensity I_{inc} is unidirectional. Therefore

$$I_{int} = 2 \times 2\pi \int_0^{\pi/2} B_{int} \cos\theta \sin\theta \, d\theta$$

$$I_{int} = 2\pi B_{int}.$$

Only a small fraction of this power per unit area will escape, since the loss cone solid angle is much less than 4π steradians. The intensity which escapes is

$$I_{esc} = 2\pi \int_0^{\theta_c} \frac{I_{int}}{2\pi} T_{esc}(\theta) \cos\theta \sin\theta \, d\theta \tag{6}$$

where $n \sin\theta_c = 1$ and θ is the internal angle of incidence. If we substitute a weighted average transmission factor $\overline{T_{esc}}$ for the angle-dependent surface transmission factor $T_{esc}(\theta)$, then the integral in (6) may be easily computed

$$I_{esc} = I_{int} \frac{\overline{T_{esc}}}{2n^2}.$$

If we now apply the principle of "Detailed Balancing," the entering intensity is made equal to the escaping intensity

$$T_{inc}(\phi) I_{inc} = I_{int} \times \frac{\overline{T_{esc}}}{2n^2}.$$

Therefore

$$I_{int} = 2n^2 \times \frac{T_{inc}(\phi)}{\overline{T_{esc}}} \times I_{inc}. \tag{7}$$

As (7) shows, the enhancement may be increased beyond $2n^2$ if $T_{inc}(\phi)$, the transmission factor into the medium, is greater than $\overline{T_{esc}}$, the average transmission factor out of the medium. Of course, time-reversal invariance guarantees that $T_{inc}(\phi) = T_{esc}(\theta)$. If the incident radiation is isotropic then the ratio $\overline{T_{inc}}/\overline{T_{esc}} = 1$ would appear in (7), ensuring that the enhancement factor is $2n^2$ as it must be for black-body radiation. For collimated radiation, an additional small enhancement $T_{inc}(\phi)/\overline{T_{esc}}$ is possible, but this comes at the expense of angular selectivity. This is fully consistent with the ordinary "Brightness Theorem" of geometrical optics [6], which states that intensity increases must come at the expense of angular selectivity. Because of the tendencies toward angular averaging described in Section II, the factor $T_{inc}(\phi)/\overline{T_{esc}}$ will be approximated as unity and, for most purposes, (7) can be rewritten

$$I_{int} = 2n^2 \times I_{inc}. \tag{8}$$

It is interesting that (8) itself could also have been derived directly from the "Brightness Theorem" [6] by taking note of the fact that the brightness defined in a medium differs from that in a vacuum simply by the factor n^2.

The intensity increases discussed thus far in this paper do not necessarily translate directly into absorption enhancements. This will be the subject of the following section.

IV. Absorption Enhancement

Two types of absorption can modify the results we have presented thus far; volume absorption in the textured optical sheet and surface absorption. In general, both types may be expected to be present. For example, in a semiconductor solar cell material, there would be absorption in the semiconductor itself and also at the surfaces due to absorption in the "transparent" electrodes, and due to imperfect reflectors at the rear surface. In this section, we will model the intensity-enhancement effects allowing for absorption. First we will set up a general method. Then we will model two specific geometries that might be of interest for solar cells.

The approach we will follow is to balance the input of light from external sources with the loss of light from the optical medium by absorption and refraction through the escape cone. The light input is $A_{inc} I_{inc} T_{inc}$, where A_{inc} is the surface area on which light is incident and the other symbols have the same meaning as before. To estimate the loss of light, we will proceed along the same lines as in Section III. We will assume that the light internal to the medium is isotropic due to the randomizing influence of refraction and reflection. There will be three contributions to light which is lost:

1) Light will escape through the escape cone at the rate

$$\frac{A_{esc} I_{int} \overline{T_{esc}}}{2n^2}$$

where A_{esc} is the surface area from which light can escape, which is not necessarily equal to A_{inc}, and the other symbols have the same meaning as before.

2) Light may be absorbed due to imperfect reflection from the boundaries

$$\int_0^{\pi/2} \eta A_{refl} I_{int} \cos\theta \sin\theta \, d\theta = \frac{\eta A_{refl} I_{int}}{2}$$

where η is the fractional absorption due to imperfect reflection at the boundaries and A_{refl} is the surface area of imperfect reflection.

3) Finally, there may be absorption within the bulk

$$\int \frac{\alpha I_{int}}{2\pi} \, dV \, d\Omega \approx \alpha l I_{int} A_{inc} \int_0^{\pi} \sin\theta \, d\theta = 2\alpha l I_{int} A_{inc} \quad (9)$$

where dV is a volume element in the bulk, α is the absorption coefficient, and (9) may be regarded as a definition of the effective thickness l. This will be approximately the mean thickness of the sheet. Equation (9) implicitly assumes that the bulk absorption is sufficiently weak that I_{int} is uniform throughout the volume.

Equating the light gained to the light lost

$$A_{inc} T_{inc} I_{inc} = \left\{ \frac{A_{esc} \overline{T_{esc}}}{2n^2} + \frac{\eta A_{refl}}{2} + 2\alpha l A_{inc} \right\} I_{int}. \quad (10)$$

Regarding I_{int} as the unknown, the expression may be rewritten

$$I_{int} = \frac{T_{inc} I_{inc}}{\{(A_{esc}/A_{inc})(\overline{T_{esc}}/2n^2) + (\eta/2)(A_{refl}/A_{inc}) + 2\alpha l\}}. \quad (11)$$

Although (11) is much more complex than (7) and (8), there are many realistic situations where the simpler expressions are adequate approximations.

One of the main questions we have in this paper is the extent to which the effects we have been discussing will act to

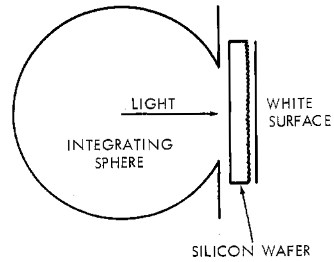

Fig. 5. The integrating sphere was used to determine the light absorbed in the silicon wafer. A comparison is made between the reflectivity of a wafer with a rough ground rear surface (as shown) and with a polished rear surface. In the former case, there was some light lost out the edges of the silicon wafer as well as by absorption.

enhance volume absorption. Using (11) the volume absorption may be written

$$2\alpha l A_{inc} I_{int} = \frac{2\alpha l A_{inc} T_{inc} I_{inc}}{\{(A_{esc}/A_{inc})(\overline{T_{esc}}/2n^2) + (\eta/2)(A_{refl}/A_{inc}) + 2\alpha l\}}.$$

The fraction of the incoming light which is absorbed in the volume is

$$f_{vol} \equiv \frac{2\alpha l A_{inc} I_{int}}{A_{inc} I_{inc}} = \frac{2\alpha l T_{inc}}{\{(A_{esc}/A_{inc})(\overline{T_{esc}}/2n^2) + (\eta/2)(A_{refl}/A_{inc}) + 2\alpha l\}}. \quad (12)$$

This reduces simply to the transmission factor T_{inc} of the incoming light in the limit of very high absorption coefficient α.

A corresponding expression may be written for the total fraction absorbed, including absorption due to imperfect reflection at the surfaces

$$f_{tot} = \frac{2\alpha l + (\eta/2)(A_{refl}/A_{inc})}{(A_{esc}/A_{inc})(\overline{T_{esc}}/2n^2) + (\eta/2)(A_{refl}/A_{inc}) + 2\alpha l} T_{inc}. \quad (13)$$

The absorption enhancement in (12) and (13) is of direct interest for weakly absorbing indirect-gap semiconductors like crystalline silicon. As (12) shows, volume absorption can be very substantial even when αl is only $1/4n^2$ which is 1/50 for silicon. The absorption enhancement factor is twice the intensity enhancement factor due to angle averaging effects. The use of these formulas is best illustrated by some specific examples.

Consider the reflectivity of a silicon wafer as measured on an integrating sphere, which is shown in Fig. 5. For these measurements, a white reflective medium was placed behind the wafer. The front surface of the wafer was polished. The main idea is to compare the overall reflectivity when the rear surface is either ground rough or polished smooth. The comparison is made in Fig. 6. With both surfaces polished, we have a plane parallel plate, the situation described in Fig. 2(b) where angular randomization within the silicon does *not* occur. The light simply makes a round trip in the wafer.

On the other hand, if the rear surface of the silicon is ground rough, internal angular randomization does occur. We may apply (12) and (13) to this situation. The Fresnel transmission of the silicon front surface T_{inc} is about 0.68. The areas A_{esc}

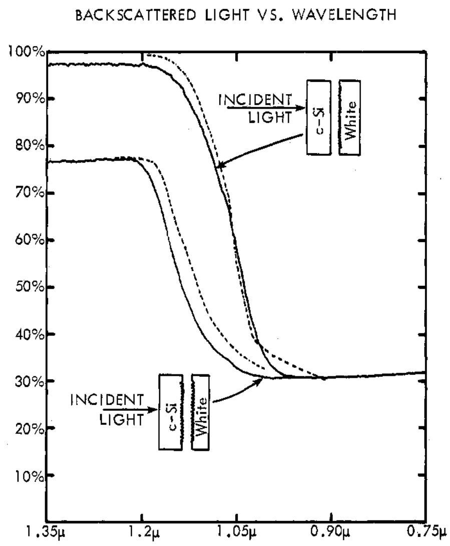

Fig. 6. The reflectivity of a silicon wafer whose rear surface is either ground rough or smoothly polished. Smooth lines are experiment. The dashed lines are theory. The theory for the rough ground surface is (13) in the text. The theory for the smooth polished surface assumes simple round-trip absorption.

and A_{inc} are the same and equal to the front surface area. The rear surface was covered with $MgCO_3$ an almost perfect white reflector, which is frequently used as a reference of whiteness. In the geometry of Fig. 5, the edges of the silicon wafer are actually external to the integrating sphere. Some of the internally trapped light, therefore, escapes across the cylindrical surface defined by the periphery of the round opening in the integrating sphere. This cylindrical surface in the silicon can be regarded as an imperfect reflector of area $A_{refl} = 2\pi rl$ where r is the radius of the opening in the integrating sphere. Therefore

$$A_{refl}/A_{inc} = 2\pi rl/\pi r^2 = 2l/r.$$

The parameters in this experiment were $r = 1$ cm and $l = 0.025$ cm. The quantity η which represents the departure from unit reflectivity at this edge is difficult to estimate *a priori*, since it depends on the details of the roughness. The value $\eta = 0.82$ describes well the wavelength-independent backscattered light in Fig. 6 in the transparent region between 1.2 and 1.35 μm. With these values for parameters and the known wavelength-dependent [7] absorption coefficient, a fairly good fit is obtained between $(1 - f_{tot})$ from (13) and experiment through the band-edge transition wavelengths (dashed and smooth lines, respectively, in Fig. 6).

The geometry described in Figs. 5 and 6 is a very favorable one for solar cells and was first described [1], [2] some time ago. Fig. 6 shows clearly the shift of the effective absorption edge toward the infrared for the light-trapping case.

Another geometry which has received some interest [8], consists of grains of silicon embedded in a glass sheet as shown in Fig. 7. It has already been recognized that the light which falls in the glass between the grains is not wasted. It tends to be trapped and eventually find its way into the silicon grains. We may analyze that situation in a similar way as previously. Let us denote the quantities pertaining to the incident light by the subscript 1, those pertaining to the silicon by the subscript 2, and those pertaining to the glass by subscript 3. Let us also assume that the white backing layer is perfectly reflective. The energy balance for the glass may be written

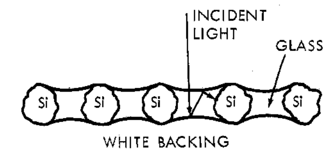

Fig. 7. A composite sheet consisting of silicon grains in a glass matrix. The theory in the text shows that even the light which falls between the grains will tend to be absorbed ultimately by the silicon.

$$\frac{A_{23}\overline{T}_{23}I_2}{2(n_2/n_3)^2} + A_{13}T_{13}I_1 = \left(\frac{A_{13}\overline{T}_{13}}{2n_3^2} + \frac{A_{23}\overline{T}_{23}}{2}\right)I_3 \quad (14)$$

where the first expression on the left-hand side is the light escaping from the silicon into the glass and the second expression is the incident light. On the right-hand side are the two terms describing the escape of light into the air and into the silicon, respectively. A similar energy balance may be written for the silicon

$$\frac{A_{23}\overline{T}_{23}I_3}{2} + A_{12}T_{12}I_1 = \left(\frac{A_{12}\overline{T}_{12}}{2n_2^2} + \frac{A_{23}\overline{T}_{23}}{2(n_2/n_3)^2} + 2\alpha l A_{12}\right) I_2. \quad (15)$$

Here there is an additional term due to absorption and l represents a typical absorption thickness of the silicon grain. Equations (14) and (15) should be regarded as two simultaneous equations in the two unknowns I_2 and I_3. A specific numerical solution would be helpful in estimating the extent to which light falling between the grains tends to be wasted. For this purpose, let us assume that the area of the sheet occupied by the silicon grains A_{12} equals the area A_{13} filled in by the glass. For numerical simplicity, let us assume also that A_{23} is also the same area. For the indices of refraction take $n_2 = 3.53$ in the silicon and $n_3 = 1.5$ in the glass. For the transmission factor $\overline{T}_{23}$ of the glass–silicon interface, take the normal incidence Fresnel reflectivity, $1 - (n_2 - n_3)^2/(n_2 + n_3)^2 = 0.84$. Let us assume there is an antireflection coating for the incident rays into the silicon so that $T_{12} \approx 0.96$ which is the same as the transmission coefficient into the glass. With these values of parameters, the simultaneous equations (14) and (15) can be solved

$$I_2 = \left(\frac{0.8316}{\alpha l + 0.0319}\right) 0.96 I_1 \quad (16)$$

where $0.96I_1$ is the transmitted incident intensity, and the quantity in parentheses is a type of enhancement factor. The important implications of (16) become apparent only upon examination of its various limits. Consider, for example, the situation in the region of wavelengths within the bandgap of silicon where $\alpha = 0$. Then

$$I_2 = \frac{0.8316 \times 0.96}{0.0319} I_1$$

which just happens to be $25I_1 = 2n_2^2 I_1$. This, of course, must be according to the considerations of Section II.

The most important parameter we are interested in is the absorption in the silicon. This may be expressed as

$$2\alpha l I_2 = \frac{1.6632\,\alpha l}{0.0319 + \alpha l} \times 0.96 I_1 \tag{17}$$

when normalized to the area of the silicon grains.

In the limit of very small αl the enhancement factor is $2n_2^2$ as before. In the limit of large αl, i.e., in the visible wavelength range for silicon

$$\lim_{\alpha \to \infty} 2\alpha l I_2 = 1.6632 \times 0.96 I_1. \tag{18}$$

According to (18), the power absorbed by the silicon exceeds the power actually falling on the silicon by a factor 1.6632 This is because the silicon collects not only those rays which it intercepts directly, but also a major fraction of those intercepted by the glass. Since only a fraction of the incident light will tend to escape the glass, the balance (≈ 0.66) will tend to be collected by the adjacent silicon grains. In other words, 83 percent of the light which enters the sheet will end up being used, in spite of the fact that the area coverage of the silicon grains is only 50 percent. If the surface area ratios were more favorable than the conservative assumptions in this calculation, then an even greater fraction of the light would end up in the silicon. The extraordinary utility of the structure described in Fig. 7 is doubly confirmed when it is also realized that because of the enhancement in the low-absorption regime, the thickness of the layer of grains can be reduced by 50. Therefore, all the advantages of a thin solar cell, mentioned in the Introduction of this paper would accrue to the structure in Fig. 7. Not the least of which is the likelihood of that structure being cheaper to fabricate than conventional silicon solar cells.

The methods described in this section are meant to suggest the approach which can be used for intensity enhancement in the presence of absorption. Refinements are still needed for the high-absorption case. The formulas given here tend to underestimate the absorption in that instance, since they assume randomization; but the absorption may be complete before randomization sets in.

In this paper, we have shown the utility of a statistical mechanical approach toward the optics of textured and inhomogeneous sheets. This work was mainly motivated by its applicability toward solar cells and other types of solar collectors. The basic enhancement factor for intensity of $2n^2$ becomes $4n^2$ for bulk absorption and n^2 for surface absorption, due to angle averaging effects. It is because many semiconductors tend to have large indices of refraction n, that these effects are particularly important in those materials.

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