# Correspondence

## Generation of a Short Optical Pulse of Arbitrary Shape and Phase Variation

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*Abstract*—It is shown that the problem of generating an optical pulse of arbitrary shape and phase may be reduced to the problem of producing an arbitrary spectral filter. This generalizes the short-pulse generation techniques, which are based on the laser breakdown switch, as the active element. We examine the extent to which any desired filter may be realized in practice, and some examples are given.

### INTRODUCTION

In recent years there has been great progress in the generation of short optical pulses by techniques that do not depend upon mode locking. Instead, they depend on the laser breakdown switch [1] which cuts off transmission with a fall time in the picosecond range. This step function is then converted into a short optical pulse by transmission through an appropriate spectral filter as shown in Fig. 1.

Among the types of optical filters which have been suggested are the Michelson interferometer [2], the Fabry–Perot etalon [3], the grating monochromator [4], and resonant-material absorbers [5]. Each of these filters produces a pulse with a characteristic shape and phase variation. In this correspondence we will turn the problem around: For a given pulse shape we will derive the specific spectral transmittance function which is required to generate it.

Thus the problem of generating an optical pulse of arbitrary shape and phase may be reduced to the problem of producing an arbitrary spectral filter. Furthermore, we will examine the extent to which any desired filter may be realized in practice.

Fig. 1 is a block diagram showing the basic concept of short-pulse generation from a laser spark. A long-pulse laser produces a breakdown plasma between a lens pair. This blocks off transmission of the beam, thereby amplitude modulating it. In the frequency domain, sidebands are associated with the amplitude modulation. The spectral filter rejects the incident laser wavelength but transmits the sidebands produced at the instant of breakdown. After the plasma has fully formed, again nothing is transmitted. Thus a step function is converted into a short pulse.

### THEORY

Let $E(t)$ represent the incident laser pulse whose duration is assumed to be much longer than any of the other times of interest in this problem. We will choose $t = 0$ as the instant at which the plasma forms, blocking off the beam. Then the electric field of the light wave transmitted through the lens pair may be represented as

$$E(t) = 1, \quad \text{for} \quad -\infty < t < 0$$

$$E(t) = 0, \quad \text{for} \quad 0 < t < \infty \tag{1}$$

i.e., a step-function amplitude modulation. This electric field

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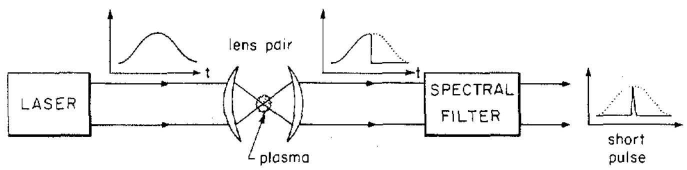


Fig. 1. A block diagram of the basic laser breakdown pulse-shaping scheme. The spectral filter rejects all but the plasma-induced sidebands on the laser pulse.

may also be written in terms of its Fourier amplitude

$$E(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} E(t) \exp(i\omega t)\, dt$$

$$E(\omega) = \frac{1}{\sqrt{2\pi i\omega}}. \tag{2}$$

(In this correspondence, the factor $\exp\{i\omega_0 t\}$ in the time dependence of the electric field will be suppressed. Frequencies $\omega$ are generally measured relative to $\omega_0$, the center frequency of the laser, unless otherwise noted.)

From (2) we see that the sidebands of the step-function amplitude modulation fall off inversely with the frequency shift from line center.

Let $G(\omega)$ represent the electric-field spectral transmittance function of the optical filter. $G(\omega)$ has both an amplitude and a phase and should be distinguished from the intensity transmission function $|G(\omega)|^2$ which has no phase information. The output electric field in the frequency domain $E_0(\omega)$ is then simply $G(\omega)\, E(\omega)$ or, using (2),

$$E_0(\omega) = \frac{G(\omega)}{\sqrt{2\pi i\omega}}. \tag{3}$$

The output pulse in the time domain is simply the inverse Fourier transform of (3).

Equation (3) determines the output field amplitude for a given spectral transmittance function. The relationship may readily be inverted

$$G(\omega) = \sqrt{2\pi}\, i\omega E_0(\omega). \tag{4}$$

This tells us the function $G$, which is required to produce a desired output pulse of Fourier amplitude $E_0(\omega)$. Equation (4) is the central result of this correspondence. It may also be written

$$G(\omega) = \int_{-\infty}^{\infty} \exp(i\omega t) \frac{dE_0(t)}{dt}\, dt \tag{5}$$

which expresses $G(\omega)$ directly in terms of the output field in the time domain $E_0(t)$.

To satisfy causality, $G(\omega)$ must be analytic in the upper half-plane. This requires $([dE_0(t)]/dt) = 0$ for $t < 0$; i.e., no change in output until the spark forms. We will make the stronger restriction $E_0(t) = 0$ for $t < 0$, thereby both satisfying causality and demanding that $G(0) = 0$. This last condition ensures that the incident laser beam does not leak through the filter in the absence of a spark.

## EXAMPLES

a) What spectral filter is required to produce a $\delta$-function pulse? A $\delta$-function pulse has Fourier amplitude $E_0(\Delta\omega) =$ constant, independent of frequency over its bandwidth. (In this section we will use the sumbol $\Delta\omega$ to distinguish relative frequency from absolute frequency $\omega$; i.e., $\Delta\omega = \omega - \omega_0$.) Using (4) the transmittance function should be

$$G(\Delta\omega) = \sqrt{2\pi}\, i\Delta\omega \times \text{constant}. \tag{6}$$

Thus we require a filter whose transmittance is linearly proportional to the frequency difference from the laser frequency. It turns out that a quarter-wave antireflection coating, when used in reflection, has precisely this behavior over a wide bandwidth (see Fig. 2)

$$\text{reflection coefficient} = \frac{E_R}{E_{\text{in}}} = \frac{1-n}{1+n}\left[1 + \exp\left\{\frac{i2nt\omega}{c}\right\}\right]. \tag{7}$$

Using the condition for zero reflection at the laser frequency, (7) becomes

$$\frac{E_R}{E_{\text{in}}} = \frac{1-n}{1+n}\left[1 - \exp\left\{\frac{i\pi\Delta\omega}{\omega_0}\right\}\right].$$

If $\Delta\omega << \omega_0$, then we may expand the exponential in a power series keeping only the first two terms

$$\frac{E_R}{E_{\text{in}}} = i\pi \frac{(1-n)}{(1+n)} \frac{\Delta\omega}{\omega_0} \tag{8}$$

which is the required functional form.

It is interesting to note in passing that a filter described by (8), or equivalently (6), is an optical differentiator. Its output is the time derivative of its input. This is proven in the Appendix.

b) What spectral filter is required to produce a rectangular pulse of duration $T$? Such a pulse has a Fourier amplitude

$$E_0(\Delta\omega) = \frac{1}{\sqrt{2\pi} i\Delta\omega}\,[\exp(i\Delta\omega T) - 1].$$

Therefore, the required spectral transmittance function is

$$G(\Delta\omega) = \sqrt{2\pi}\, i\Delta\omega E_0(\Delta\omega) = [\exp(i\Delta\omega T) - 1]. \tag{9}$$

A Michelson interferometer [2], used as shown in Fig. 3, and adjusted for destructive interference of the output beam, satisfies (9). In particular

$$\frac{E_0}{E_{\text{in}}} = [1 - \exp(i\omega T)] = [1 - \exp(i\omega_0 T)\exp(i\Delta\omega T)].$$

Destructive interference at the laser frequency requires $\exp\{i\omega_0 T\} = 1$, where $T$ is the difference in optical path between the two arms of the interferometer. Therefore, the Michelson interferometer satisfies (9).

c) What spectral filter is required to produce an exponentially decaying pulse of time constant $\tau$? Such a pulse has a Fourier amplitude

$$E_0(\Delta\omega) = \frac{\tau}{\sqrt{2\pi}\,(1 - i\Delta\omega\tau)}.$$

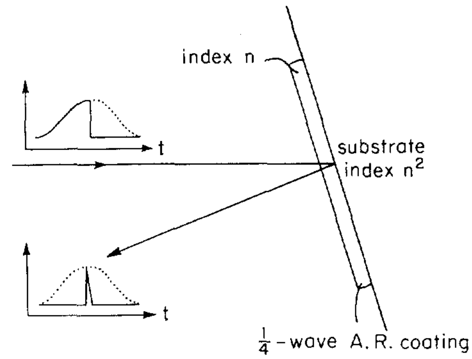


Fig. 2. A quarter-wave antireflection coating, when used in reflection, acts as an optical differentiator (see Appendix). It converts the step function into a $\delta$-function pulse.

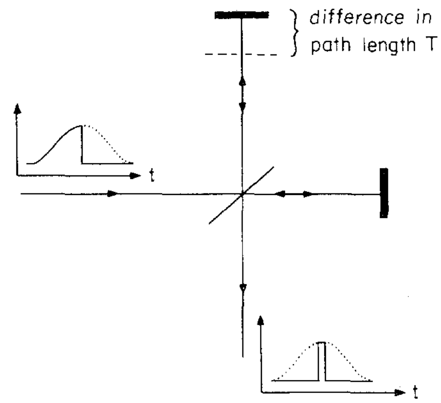


Fig. 3. A Michelson interferometer is used to generate a rectangular pulse of duration $T$ which is the difference in path length between the two arms of the interferometer.

Therefore, the required spectral transmittance function is

$$G(\Delta\omega) = \sqrt{2\pi}\, i\Delta\omega E_0(\Delta\omega) = \frac{i\Delta\omega\tau}{1 - i\Delta\omega\tau}. \tag{10}$$

Such an optical filter may be realized as shown in Fig. 4. It requires two filter elements in series, the $G(\Delta\omega)$ of the combination being the product of the individual $G$'s. One element is simply an optical differentiator (see the Appendix), such as the quarter-wave antireflection coating which supplies the factor $i\Delta\omega$ to the numerator. The other element is a Michelson interferometer, this time adjusted for equal-length arms. In one of the arms is an optically thin, homogeneously broadened, resonant absorber. It is readily shown that such an element contributes the factor $(1 - i\Delta\omega\tau)$ in the denominator of (10). The homogeneous linewidth of the absorber is $1/\tau$. If it is off-resonance, then an oscillatory decaying pulse is produced. A somewhat different pulse shape was recently generated by using an optically thick resonant medium as the filter [5].

## DISCUSSION

The previously given examples are meant to indicate the generality of the pulse-shaping approach being proposed here. Since spectral filters are linear elements, many of them may be combined in series to produce the required transmittance function. There is great versatility in using media, both resonant and nonresonant, as filtering elements.

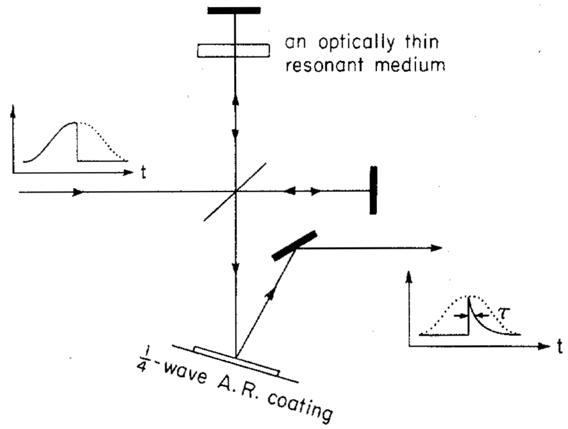


Fig. 4. A composite filter element, consisting of a Michelson interferometer and a quarter-wave antireflection coating used in series. An optically thin resonant medium of homogeneous linewidth $1/\tau$ unbalances one of the arms of the interferometer. The output is an exponentially decaying pulse of time constant $\tau$.

A type of filter which was not mentioned in the previously given examples, but which has great potential, is the multilayer dielectric-coated interference filter. These are now mainly used in bandpass applications, for which only $|G(\omega)|^2$ is important. If designed to take account of both the phase and amplitude of $G(\omega)$, they could be used in pulse-shaping applications. It should be possible to closely approximate any given function by using enough layers. Complicated pulse shapes, such as those which have been proposed for laser fusion [6], may require computer design to optimize the layer spacing and other parameters.

## Practical Limitations

There are, of course, practical limitations to these pulse-generation techniques. For example, many of the filters which we have been discussing have the property of zero transmission at the center laser wavelength; i.e., $G(0) = 0$. This is important for rejecting the laser light at $t < 0$ before the breakdown spark has formed. In practice, a perfect null is not always possible and the rejection may not be 100-percent complete.

Another limitation is related to the finite fall time $T$ of the light as it passes through the breakdown plasma. Equation (1) assumes an instantaneous fall time, which is only approximate. In particular, (2) is correct only for frequency shifts $\Delta\omega \lesssim 1/T$. Therefore, the techniques described in this correspondence are useful for generating pulses whose overall frequency spectrum is no wider than $2/T$. Understandably, it is not possible to generate pulses which are shorter than the fall time $T$. Recent measurements indicate that $T$ is in the picosecond range for $CO_2$-laser-produced plasmas. There is speculation that even faster fall times are possible.

## Conclusion

We have shown that the problem of generating an arbitrary optical pulse $E(t)$ may be reduced to the problem of producing a matched filter as given by (5). Some examples of specific pulse shapes and their matching filters were also presented.

## Appendix

We wish to show that a filter whose spectral transmittance function is described by (6) acts as an optical differentiator. By definition

$$E_{\text{out}}(\omega) = \sqrt{2\pi}\, i\omega E_{\text{in}}(\omega).$$

Taking the inverse Fourier transform

$$\begin{aligned} E_{\text{out}}(t) &= \int_{-\infty}^{\infty} i\omega \exp(-i\omega t)\, E_{\text{in}}(\omega)\, d\omega \\ &= -\frac{d}{dt}\int_{-\infty}^{\infty} \exp(-i\omega t)\, E_{\text{in}}(\omega)\, d\omega \\ &= -\frac{d}{dt} E_{\text{in}}(t). \end{aligned}$$

Therefore, the output of such a filter is the time derivative of its input. Several such filters may be used in series to obtain higher order derivatives.

## References


[1] E. Yablonovitch, "Self-phase modulation and short-pulse generation from laser-breakdown plasmas," *Phys. Rev.*, vol. A10, pp. 1888–1895, Nov. 1974.

[2] a) A. Szoke, J. Goldhar, H. P. Grieneisen, and N. A. Kurnit, "Generation of variable length laser pulses in the subnanosecond region," *Opt. Commun.*, vol. 6, pp. 131–134, Oct. 1972.
b) D. Milam *et al.*, "Production of intense subnanosecond pulses by cavity dumping," *IEEE J. Quantum Electron.*, vol. QE-10, pp. 20–25, Jan. 1974.

[3] M. Duguay, private communication.

[4] a) E. Yablonovitch, "Spectral broadening in the light transmitted through a rapidly growing plasma," *Phys. Rev. Lett.*, vol. 31, pp. 877–879, Oct. 1973.
b) ——, "Self-phase modulation of light in a laser-breakdown plasma," *Phys. Rev. Lett.*, vol. 32, pp. 1101–1104, May 1974.

[5] E. Yablonovitch and J. Goldhar, "Short $CO_2$ laser pulse generation by optical free induction decay," *Appl. Phys. Lett.*, vol. 25, pp. 580–582, Nov. 1974.

[6] J. Nuckolls, L. Wood, A. Thiessen, and G. Zimmerman, "Laser compression of matter to super-high densities: Thermonuclear (CTR) applications," *Nature*, vol. 239, pp. 129–142, Sept. 1972.


# Excitation of Cold Cavity Modes in a Coupling-Modulated $CO_2$ Laser

A. L. SCHOLTZ

*Abstract*—**The amplitudes of cold longitudinal cavity modes excited as an unwanted side effect of coupling modulation were studied in a $CO_2$ laser. Experimental observations agree well with the results of an analytical approach treating the effect of coupling modulation on the cavity field as loss modulation.**

## Introduction

Intracavity coupling modulation [1] is an efficient method to modulate $CO_2$ lasers [2]–[4]. With this modulation scheme, the unwanted excitation of cold cavity modes is of particular interest [5], [6]. Cold modes are longitudinal modes of the optical resonator which do not coincide with the laser transition. They can be excited by any kind of intracavity modulation if the modulating frequency $f_m$ is approximately an integer multiple of the cavity mode spacing $c/2l$ ($l$

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