Complex Layered Materials and
Periodic Electromagnetic Band-Gap Structures:
Concepts, Characterizations, and Applications

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by

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To my wife, Afsaneh …

whose love, patient, and support made it all possible.
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PUBLICATIONS AND PRESENTATIONS

H. Mosallaei, and Y. Rahmat-Samii, “Grand challenges in analyzing EM band-gap
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The main objective of this dissertation is to characterize and create insight into the electromagnetic performances of two classes of composite structures, namely, complex multi-layered media and periodic Electromagnetic Band-Gap (EBG) structures. The advanced and diversified computational techniques are applied to obtain their unique propagation characteristics and integrate the results into some novel applications.

In the first part of this dissertation, the vector wave solution of Maxwell’s equations is integrated with the Genetic Algorithm (GA) optimization method to provide a powerful technique for characterizing multi-layered materials, and obtaining their optimal designs. The developed method is successfully applied to determine the optimal composite coatings for Radar Cross Section (RCS) reduction of canonical structures.
Both monostatic and bistatic scatterings are explored. A GA with hybrid planar/curved surface implementation is also introduced to efficiently obtain the optimal absorbing materials for curved structures. Furthermore, design optimization of the non-uniform Luneburg and 2-shell spherical lens antennas utilizing modal solution/GA-adaptive-cost function is presented. The lens antennas are effectively optimized for both high gain and suppressed grating lobes.

The second part demonstrates the development of an advanced computational engine, which accurately computes the broadband characteristics of challenging periodic electromagnetic band-gap structures. This method utilizes the Finite Difference Time Domain (FDTD) technique with Periodic Boundary Condition/Perfectly Matched Layer (PBC/PML), which is efficiently integrated with the Prony scheme. The computational technique is successfully applied to characterize and present the unique propagation performances of different classes of periodic structures such as Frequency Selective Surfaces (FSS), Photonic Band-Gap (PBG) materials, and Left-Handed (LH) composite media. The results are incorporated into some novel applications such as high $Q$ nanocavity lasers, guiding the electromagnetic waves at sharp bends, and miniaturized microstrip patch antennas.
Chapter 1

Introduction

Many of novel technological designs have resulted by analyzing the properties of materials and proposing new structural configurations for them. Multi-layered and periodic dielectric/metallic structures are two types of composite media with the potential applications in Electromagnetics (EM). Utilizing these materials one can effectively control the propagation of EM waves and present the novel designs. In order to develop a new structural configuration with unique properties, one needs to thoroughly understand the characteristics of the structure. This can be accomplished by applying an advanced computational engine.

The materials presented in this dissertation will provide a comprehensive description of two classes of structures, namely, complex multi-layered media and periodic Electromagnetic Band-Gap (EBG) structures. The objective is to analyze theoretically/numerically their interaction with electromagnetic waves, identify their innovative propagation characteristics, and incorporate the results into some novel applications.

To accomplish this, two potentially powerful computational techniques, as outlined below, are developed.
(a) Vector wave solution of Maxwell’s equations integrated with the Genetic Algorithm (GA) optimizer for characterizing layered composite materials,

(b) Finite Difference Time Domain (FDTD) technique with Periodic Boundary Condition/Perfectly Matched Layer (PBC/PML) integrated with the Prony scheme to efficiently construct the broadband performance of the periodic band-gap structures.

The developed computational engines are successfully incorporated into the following areas of interest

- Radar Cross Section (RCS) reduction of canonical targets,
- Non-Uniform Luneburg and 2-Shell lens antennas,
- Frequency Selective Surfaces (FSS),
- Photonic Band-Gap (PBG) materials and their potential applications,
- Composite media with negative permittivity/permeability.

In the following sections, each of the above-itemized areas of research is briefed with the research goals of each listed.

1.1 **Optimal Composite Materials for RCS Reduction of Canonical Targets and Design of High Performance Lens Antennas**

The focus is to integrate the modal solution of Maxwell’s equations with the Genetic Algorithm (GA) to present the optimal composite materials for two classes of applications namely, (a) Radar Cross Section (RCS) reduction of canonical targets, and (b) design optimization of the non-uniform Luneburg and 2-shell lens antennas. The analysis technique and design optimization of the layered structures are addressed.
1.1.1 Vector Wave Solution/GA Technique

To obtain the electromagnetic performance of complex multi-layered materials such as planar, cylindrical, or spherical structures, the vector wave solution of Maxwell’s equations is applied to accurately determine the modal solution in their corresponding coordinate systems [1]-[3].

Next the modal solution is integrated with the GA technique to present the optimal composite layered structure. The GA technique is a global optimizer, and is very efficient in optimizing the new electromagnetic problems having discontinuities, constrained parameters, discrete solution domains, and a large number of dimensions with many local optima [4].

An accurate and capable computer code is developed to integrate the modal solution with the GA technique effectively. Fig. 1.1 shows the schematic of the computational engine.

1.1.2 RCS Reduction of Canonical Targets

Radar cross section reduction of a target using multi-layered Radar Absorbing Materials (RAM) is an important consideration in radar systems [5]. The properties of the RAM depend on the frequency and for wide-band absorption one needs to obtain a proper composite selection of these materials.

In this research, the modal solution/GA technique, as depicted in Fig. 1.2, is effectively applied to analyze the layered planar, cylindrical, and spherical structures, and successfully obtain the optimal wide-band absorbing coatings for the canonical targets.
Both monostatic and bistatic RCS reduction of the structures are investigated. The effectiveness of the optimal planar coating in reducing RCS of the curved surfaces is also studied. Furthermore, a GA with hybrid planer/curved surface population initialization is introduced to efficiently design the optimal composite coatings with remarkable reduction of the RCS of arbitrary curved structures.

1.1.3 Design Optimization of the Non-Uniform Luneburg and 2-Shell Lens Antennas

Multi-shell spherical lens antennas have recently regained interest for beam scanning at millimeter and microwave frequencies in mobile and satellite communication systems [6]. The spherical lens transforms the point source radiation into the plane wave by modifying its phase distribution. Since the mathematical principle of the lens is based on the geometrical optics concept, it can typically operate over a broad band of frequency. On the other hand, spherical symmetry of the lens allows for multi-beam scanning application by placing an array of feeds around the lens antenna.

Among various lenses, the Luneburg lens has received much attention. An ideal Luneburg lens is a spherical lens antenna with continuous permittivity from 2 at the center to the 1 at the outer surface. However, in practice, the lens is constructed from the multi-layered uniform spherical shells, which may degrade the performance of the lens by generating grating lobes and reducing the gain of the antenna.

The focus of this work is to present an optimal non-uniform lens antenna with low number of spherical shells and high performance radiation characteristics. To this end,
the GA with adaptive cost function is integrated with the modal solution of Maxwell’s equations to obtain the optimal material and thickness of the spherical shells for achieving both high gain and low sidelobe level (Fig. 1.3). Design optimization of various lens geometries including air-gap and feed offset from the lens is studied. Additionally, a novel optimized 2-shell lens antenna with almost the same performance as the Luneburg lens and with low side lobe envelope variations is presented. Many useful engineering guidelines are highlighted.

Fig. 1.1: Vector wave solution of Maxwell’s equations integrated with the genetic algorithm optimizer for designing optimal composite materials.
Fig. 1.2: Optimal RAM coatings for RCS reduction of canonical targets.

Fig. 1.3: Design optimization of the non-uniform lens antennas.
1.2 Broadband Characteristics of Periodic Electromagnetic Band-Gap Structures

Periodic structures have numerous applications in the design of novel configurations in electromagnetics, such as Frequency Selective Surfaces (FSS) and Photonic Band-Gap (PBG) materials. In this dissertation these structures are classified under the broad terminology of “Electromagnetic Band-Gaps (EBG)” [7]. To investigate the potential applications of the periodic band-gap structures, one needs to effectively characterize their interactions with the electromagnetic waves.

The main objective of this research is to provide a powerful computational engine based on the FDTD/Prony technique to accurately analyze the complex periodic structures in layered inhomogeneous media, and then determine their unique electromagnetic characteristics. The results are used to understand the physical phenomena of these useful structures, and applied to obtain some of their novel applications.

1.2.1 FDTD Computational Engine

In this research, to present the broadband performance of various complex periodic structures a very powerful computational technique utilizing FDTD with PBC/PML boundary conditions is developed [8], [9]. The split-field approach is applied to discretize the Floquet transformed Maxwell’s equations. Furthermore, the Prony extrapolation scheme is integrated to increase the efficiency of the technique. It is demonstrated that the developed technique is a very capable engine in characterizing different classes of
challenging periodic band-gap structures. The main parts of the computational engine are briefed in Fig. 1.4.

1.2.2 Frequency Selective Surfaces

Frequency selective surfaces composed of complex periodic scatterers of dielectric and conductor of arbitrary shapes have numerous applications in electromagnetics [10], [11]. These structures can totally reflect the EM waves in some ranges of frequency, while they can be completely transparent to the EM waves in other ranges. The shape of the resonance element itself and the lattice geometry in which the elements are arranged are the main factors controlling the performance of the periodic structures. Notice that the FSS typically cover limited angle of arrival and they are sensitive to the polarization states.

In this work, the FDTD/Prony technique is successfully applied to determine the electromagnetic propagation characteristics of different types of complex and challenging FSS structures (Fig. 1.5) such as, (a) double concentric square loop FSS, (b) dipole FSS manifesting high $Q$ resonance, (c) crossed dipole FSS, (d) dichroic plate FSS, (e) inset self-similar aperture FSS, (f) fractal FSS, (g) multi-layered tripod FSS, (h) 9-layer dielectric/inductive square loop FSS, and (i) smart surface mushroom FSS with reduced edge diffraction effects. The results are extremely satisfactory demonstrating the capability and accuracy of the developed engine.
1.2.3 Photonic Band-Gap Materials

Photonic band-gap structures are typically a class of periodic dielectric materials, which by generating an electromagnetic band-gap forbid the propagation of the electromagnetic waves [12]. The discovery of the PBG structures has created unique opportunities for controlling the propagation of EM waves, leading to numerous novel applications in the optical and microwave technologies.

In this research, a comprehensive treatment of these useful band-gap structures in controlling the propagation of EM waves is presented. The FDTD/Prony technique is applied to obtain the band-gap phenomena of three classes of periodic dielectric materials, namely, triangular, rectangular, and woodpile PBG structures, as displayed in Fig. 1.6. In this manner, the reflection coefficient of the plane wave incident on the band-gap structure is computed. Compared to the dispersion diagram method, which is usually applied to analyze these structures, the presented technique appears to have two potential advantages as, (a) obtaining performance of the structure outside the gap region, and (b) presenting phase and polarization behaviors of the band-gap structure.

The concept of PBG structures, constructed from isolated dielectric columns or connective dielectric lattice, in generating the TE or TM band-gap regions for in-plane incident waves is also investigated. Furthermore, the possibility of modeling the performance of PBG periodic structures utilizing the effective dielectric materials is explored.
The unique characteristics of the PBG dielectric materials in controlling the propagation of EM waves are incorporated into the three potential applications illustrated in Fig. 1.7:

- **High Q dielectric nanocavity lasers**: Periodic PBG/total internal reflection is used to effectively localize the EM waves in three directions.

- **Guiding the EM waves in sharp bends**: An array of the PBG holes in the guiding direction is removed to successfully channel the EM waves. A 60° shaped bend is also introduced to improve the coupling of the waves in tight turns.

- **Miniaturized microstrip patch antennas**: PBG material is integrated to suppress the surface waves and present a miniaturized microstrip patch antenna with high performance radiation characteristics. The effectiveness of the dielectric PBG in suppression the surface waves, compared to the effective dielectric material, is also addressed.

### 1.2.4 Composite Media with Negative Permittivity/Permeability

The challenge in this work is to present a new composite material with simultaneously negative permittivity/permeability [13]. This new class of structure is called the Left-Handed (LH) material, and it has unique electrodynamic properties such as, reversal of the Doppler shift, anomalous refraction behavior, and reversal of radiation pressure to radiation tension. These phenomena can never be observed in naturally occurring materials or composites.
The composite material is constructed from two periodic structures, namely, conducting straight wires/split ring resonators, as illustrated in Fig. 1.8. The FDTD/Prony technique is applied to characterize the complex structure. It is demonstrated that the negative effective permittivity of the straight wires is properly combined with the negative effective permeability of the split ring resonators to generate a pass band through the gap regions of the periodic structures (straight wires/split rings), or to produce the LH material. The novel characteristics of the LH media may be incorporated into some future applications.

Fig. 1.4: Schematic of the FDTD/Prony computational engine for characterizing challenging EBG structures.
Fig. 1.5: Different classes of FSS for various EM applications.
Fig. 1.6: Different classes of PBG for various EM applications.
Fig. 1.7: Potential applications of the PBG structures; high $Q$ nanocavity lasers, guiding the EM waves in sharp bends, and miniaturized microstrip patch antennas.
Fig. 1.8: Composite LH material with simultaneously negative permittivity/permeability.
Chapter 2

Vector Wave Solution of Maxwell’s Equations Integrated with the Genetic Algorithm

Multi-layered complex media have tremendous applications in the different areas of electromagnetics. The accurate analysis of these layered structures [1]-[3] leads to the better understanding of their propagation characteristics, which may result in the design of new structural configurations.

The main objective of this chapter is to provide a powerful computational engine to (a) accurately characterize the multi-layered canonical structures, and (b) integrate the results with a capable optimization technique in order to present the optimal composite layered-materials incorporating into the potential applications.

To this end, the vector wave solution of Maxwell’s equations is applied to obtain the propagation characteristics of the multi-layered canonical media, namely, planar, cylindrical, and spherical structures. Next, the modal solution is integrated with the Genetic Algorithm (GA) optimization technique [4] to present the optimal composite structure. The developed vector wave solution/GA optimizer is a very valuable engine to successfully present the optimal multi-layered materials for the novel structural designs.
2.1 Vector Wave Solution of Maxwell’s Equations

In this section, the formulations for the scattering of the electromagnetic fields from the canonical structures are briefed. Fig. 2.1 depicts the geometry of the $M$-layered planar, cylindrical, and spherical structures. The total electric field in the presence of the multi-

![Multi-layered canonical structures and their corresponding coordinates.](image)

The total electric field in the presence of the multi-layered structure is written as

$$\mathbf{E} = \mathbf{E}^i + \mathbf{E}^s$$  \hspace{1cm} (2.1)

where $\mathbf{E}^i$ and $\mathbf{E}^s$ are the incident and scattered field, respectively. The scattered field $\mathbf{E}^s$ for these canonical structures is obtained using the vector wave solution of the Maxwell’s equations.

For the multi-layered planar structure, the scattered field $\mathbf{E}^s$ in the $i^{th}$ region is determined using the plane wave representation of the electromagnetic fields in the following form [1]

$$\mathbf{E}^s_i(\mathbf{r}) = \hat{\gamma}(A_i e^{-\beta z} + B_i e^{+\beta z})$$  \hspace{1cm} (2.2)
where \( k_i \) is the complex propagation constant of region \( i^{th} \). Then the unknown coefficient \( A_i \) and \( B_i \) are determined by applying the boundary conditions at the material interfaces.

The scattered electric field \( \mathbf{E}' \) for the multilayered cylindrical structure is determined by a cylindrical wave expansion of EM fields and mode matching technique [2]. For a TMz mode the scattered electric field in the \( i^{th} \) region is written as

\[
\mathbf{E}'_i(r) = \hat{\mathbf{z}} \sum_n \left( A_n J_n(k_i \rho) + B_n H_n^{(2)}(k_i \rho) \right) e^{in\phi}
\]

where \( J_n(k_i \rho) \) and \( H_n^{(2)}(k_i \rho) \) are the first and second kind cylindrical Bessel and Hankel functions, respectively, and \( k_i \) is the complex propagation constant in the \( i^{th} \) region. The expansion coefficients \( A_n \) and \( B_n \) are obtained by applying the boundary conditions. Similarly, the solution for TEz case can be determined.

In the multilayered spherical structure, the divergenceless scattered field \( \mathbf{E}' \) in the \( i^{th} \) region is represented as the summation of \( M \) and \( N \) spherical vector wave functions as [3], [14], [15]

\[
\mathbf{E}'_i(r) = \sum_{n,m} \mathbf{C}_{i, nm} \cdot \mathbf{W}_{nm}^{T}(k_i, r)
\]

where \( T \) is the transpose operator, \( \mathbf{C}_{i, nm} \) and \( \mathbf{W}_{nm}(k_i, r) \) are the expansion coefficients and spherical vector wave functions matrices in the following form

\[
\mathbf{C}_{i, nm} = \begin{pmatrix} a_{i, nm} & b_{i, nm} & c_{i, nm} & d_{i, nm} \end{pmatrix}
\]

\[
\mathbf{W}_{nm}(k_i, r) = \begin{pmatrix} M_{nm}^{(1)}(k_i, r) & M_{nm}^{(4)}(k_i, r) & N_{nm}^{(1)}(k_i, r) & N_{nm}^{(4)}(k_i, r) \end{pmatrix}
\]
The vector wave functions $M$ and $N$ are defined based on the exponential form of the $\phi$ angle, the superscripts (1) and (4) show that the vector wave functions include the first and second kind spherical Bessel and Hankel functions, respectively, and $k_i$ is the complex propagation constant of the $i^{th}$ region. The unknown coefficient matrix $C_{i,nm}$ is determined by applying the boundary conditions at the material interfaces.

Based on these formulations and by developing an accurate computer code the electromagnetic fields for these canonical structures are obtained. The computer program is a double precision code, which uses an efficient Bessel and Hankel functions computation [16], [17] for accurate determination of the scattered fields. The program internally checks the convergence of the EM fields and selects the required number of the summation terms in (2.3) and (2.4). Its accuracy has been tested against numerous available published data [18].

In the case of eccentric cylindrical or spherical structures, the EM fields in the coordinate of each cylinder or sphere is similarly obtained using the above formula. Next, the boundary conditions are imposed using the translational addition theorems for the vector cylindrical or spherical wave functions, and then the unknown coefficients are determined successfully. The formulations for the eccentric spherical structure are summarized in Appendix A.
2.2 Genetic Algorithm Optimization Technique

The genetic algorithms are iterative optimization procedures that typically start with a randomly selected population of solution domain, and gradually evolve toward better solution through the application of genetic operators that are selection, crossover, and mutation [4]. GA’s are global optimizers and have some unique distinctions with respect to the local techniques such as conjugate-gradient and quasi-Newton methods.

The local techniques are typically highly dependent on the starting point or initial guess, while the final result in global methods are largely independent of the initial starting point. Generally speaking, local techniques tend to be tightly coupled to the solution domain, resulting in relatively fast convergence to a local maximum. However, this tight solution-space coupling also places some constraints on the solution domain, such as differentiability and continuity that can be hard or even impossible to deal with in practice.

The global techniques, on the other hand, are largely independent of the solution domain. Additionally, for optimization on the discrete domain the genetic algorithm is the most efficient technique that can obtain the best set of parameters between the discrete sets of data. Consequently, the GA method, being a global optimizer, could be an efficient technique for optimizing the new electromagnetic problems having discontinuities, constrained parameters, discrete solution domain, and a large number of dimensions with many local optima [19]-[21].
In order to apply the GA method to the optimization of multi-layered structures (planar, cylindrical, and spherical structures), material and thickness of \( j^{th} \) layer are represented in the finite sequences of binary bits as

\[
L_j = M_j T_j = [m_j^1, m_j^2, \ldots, m_j^{N_{mm}}, t_j^1, t_j^2, \ldots, t_j^{N_{mm}}].
\]

(2.6)

The entire structure is subsequently represented by the sequence \( G = L_1 L_2 \ldots L_M \) as called individual. In the process of the GA implementation, first a population of individuals with size \( N_{pop} \) (in this work \( N_{pop} = 100 \)) is chosen. Next, a proper object or fitness function \( F \) is defined to assign a fitness value to each of the individuals. The genetic algorithm then proceeds by iteratively generating a new population from the previous one through the application of the genetic operators, namely, selection, crossover, and mutation.

**Selection:** During the selection operation, a new generation is derived from the existing generation using a procedure, which is referred to as tournament selection depicted in Fig. 2.2. In this process, a sub-population of \( N \) individuals is chosen at random from the general population. The one with the highest fitness is selected and the rest are placed back into the general population. This procedure is repeated until a new set of \( N_{pop} \) individuals is produced. Notice that in the binary tournament selection \( N \) is equal to two. The selection process ensures the new population to contain, on the average, more sequences with high fitness values.
Crossover: The purpose of crossover in the GA technique is to rearrange the binary bits in the individuals, with the objective of producing better combinations with higher fitness values. To this end, in a single-point crossover operation as used in this work, a pair of individuals is selected as parents, if probability $p > p_{cross}$, a random location in the parent’s chromosomes is selected. The portions of the chromosomes preceding the selected point are copied from parent number 1 and parent number 2 to child number 1 and child number 2 (new individuals), respectively. But, the chromosomes following the randomly selected point in parent number 1 and parent number 2 are placed in the corresponding locations in the child number 2 and child number 1, respectively. This procedure has been illustrated in Fig. 2.3. If $p < p_{cross}$, the entire chromosome of parent number 1 is copied into child number 1, and similarly for parent number 2 and child number 2. Typically, a crossover with probability $p_{cross} = 0.6 – 0.8$ is optimal.
Fig. 2.3: Single-point crossover operation in the GA optimizer.

**Mutation:** The mutation operator provides a means for exploring the portions of solution domain that are not represented in the currently GA population. In mutation, if \( p > p_{mut} \), a bit in the individual is randomly selected and inverted, as shown in Fig. 2.4. Generally, mutation should occur with a low probability, usually on the order of \( p_{mut} = 0.01 - 0.1 \).

Fig. 2.4: Mutation operation in the GA optimizer.

In the procedure of creating new population the elitism is also employed. Saving and inserting the best individual from the last generation is known as the elitist strategy, or simply elitism.

The new populations will increasingly contain better sequences, and eventually converge to the optimal population consisting of optimal sequences. Tracking the performance of the best sequence in the population, as well as the average performance...
of all sequences checks the convergence of the algorithm. If no improvement in both quantities in a large number of generations occurs, the procedure is assumed to have converged. The flowchart of the optimization procedure is shown in Fig. 2.5.

Fig. 2.5: Flowchart of the modal solution/GA optimization procedure.

2.3 Modal Solution/GA in Presenting Novel Designs

In this chapter, a powerful computational engine utilizing vector wave solution of Maxwell’s equations integrated with the genetic algorithm optimization technique is developed to present the optimal composite materials for the applications of interest. The computational engine is very capable for characterization layered complex structures.

The presented technique is successfully applied in the following chapters (Chapters 3 and 4) to obtain the propagation characteristics and optimal designs for two classes of applications as (a) composite RAM for reducing RCS of canonical structures, and (b) non-uniform Luneburg and 2-shell lens antennas.
Chapter 3

RCS Reduction of Canonical Targets using GA Synthesized RAM

Radar cross section reduction of a target using multi-layered radar absorbing materials has been an important consideration in radar systems [5], [22]-[24]. In complex structures, for example a fighter plane, one can identify canonical features composed of planar, cylindrical, and spherical surfaces as shown in Fig. 3.1. In designing RAM, it becomes important to obtain the optimized RAM for reducing the RCS of these canonical structures in a wide-band frequency range. It is also of interest to investigate the usefulness of the optimal planar RAM for RCS reduction in curved structures. These issues are addressed in this research by utilizing the power of mode matching technique integrated with a novel utilization of the efficiently implemented GA optimizer. Fig. 3.1 provides the key features of this implementation.

Vector wave solution of Maxwell’s equations for the multi-layered coated canonical targets, as observed in Fig. 3.1, is used as the main computational engine for the accurate and efficient computations of scattered fields. Next, the genetic algorithm optimization technique is applied to obtain the optimal RAM for reducing RCS of coated structures.
Michielssen et al. in 1993 [22] have investigated the optimal design of coating material for reducing the reflection coefficient of planar structures. The present work extends their methodology to reduce the RCS of curved surfaces, including cylindrical and spherical structures, and it attempts to provide useful design guidelines and physical observations.

Since, in general, properties of the RAM depend on the frequency, for wide-band absorption, a proper composite selection of these materials is necessary. The GA is an effective optimizer to obtain the best combination of the RAM among available database of materials. The GA is successfully applied to the synthesis of wide-band absorbing coating in a specified frequency range for reducing monostatic RCS of canonical structure. The reduction in bistatic RCS of coated conducting cylindrical and spherical structures is also investigated. Furthermore, the effectiveness of the optimal planar coating for RCS reduction in a structure with arbitrary curvature is demonstrated. This observation was essential in introducing a novel efficient GA implementation using as part of its initial generation the best population obtained for the planar RAM design.
Fig. 3.1: Flowchart of the genetic algorithm integrated with the vector wave solution of Maxwell's equations for obtaining optimal RAM. Note the possibility of using GA planar[curved surface implementation for initializing the population. Coated planar, cylindrical and spherical targets with their appropriate coordinate systems are also shown.
3.1 Vector Wave Solution/GA Implementation

In this section, the vector wave solution of Maxwell’s equations is properly integrated with the GA optimizer to present a powerful computational tool for designing optimal wide-band composite radar absorbing materials.

Fig. 3.1 depicts the $M$-layered coated conducting planar, cylindrical, and spherical structures, illuminated by a normally incident plane wave. The choice of the normal incidence was primarily made in order to provide a common base for comparing the results of various target shapes. The scattered fields and then monostatic RCS of these canonical structures are accurately obtained using the modal solutions.

The evaluated RCS’s are incorporated into the genetic algorithm optimizer to present the optimal RAM. In the context of obtaining $M$-layered wide-band absorbing coating for reducing RCS of a conducting structure, the objective or fitness function $F$ is defined by

$$
F(M_1, T_1, M_2, T_2, \ldots, M_M, T_M) = \min \left[ \text{RCS}^{(\text{dB})}_{\text{conductor}} - \text{RCS}^{(\text{dB})}_{\text{coated}} \right]_{(f_1, f_2, \ldots, f_{\Delta f})} \quad (3.1)
$$

where $M_j$ and $T_j$ represent the material choice and thickness of the $j^{th}$ layer, respectively. Application of (3.1) as a fitness function for GA is an attempt to maximize the minimum difference between the RCS (in dB) of the original target and the RAM coated one in the desired frequency band. In this study, a small database containing 16 different materials with permittivities $\varepsilon_j(f)$ and permeabilities $\mu_j(f)$, as shown in Table 3.1, is considered [22]. The design goal is to determine an optimal $M$-layered
coating in order to maximize the minimum RCS reduction in a prescribed range of frequencies \( \{f_1, f_2, \ldots, f_{N_f}\} \).

In order to apply the GA method to obtain the optimal coating, material and thickness of \( j^{th} \) layer are represented in the finite sequences of binary bits as

\[
L_j = M_j T_j = \left[ m_j^1 m_j^2 \ldots m_j^{N_mb} \right] \left[ t_j^1 t_j^2 \ldots t_j^{N_mb} \right].
\]  

The entire structure is subsequently represented by the sequence \( G = L_1 L_2 \ldots L_M \). The database includes 16 different available materials that can be represented by 4 binary bits or \( N_{mb} = 4 \). In addition, for designing the thickness of each layer with 3-digit resolution, each layer is represented by 10 binary bits with \( N_{tb} = 10 \).

For certain applications, the coating should not only absorb the incident wave in a wide range of frequency, but should also be as light as possible. Therefore, in the cases studied in this work, 5 layer coating \( (M = 5) \) with thickness \( 0 \leq T_j (cm) \leq 0.2 \) is used. The genetic algorithm is successfully applied to the synthesis of a wide-band coating in the typical frequency range \( 0.2 \leq f(GHz) \leq 2 \) by sampling the cost function \( F \) at 10 different frequency points, \( N_f = 10 \).

Based on our experience in obtaining the optimal RAM for the cylindrical and spherical structures, about 100 generations containing 100 individuals are necessary. Additionally, the RCS for each individual has to be computed at 10 different frequency points, and thus the procedure is time consuming. A GA implementation with a particular selection of initial population has proven to provide rapid convergence. In this implementation, the best population for the planar coating initializes the GA process for
the optimal RAM design of curved structures. Section 3.4 will highlight some of the unique features of this implementation.

Table 3.1: Relative permittivities and permeabilities of the 16 materials in the database [22].

<table>
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<tr>
<th>#</th>
<th>$\varepsilon_r$</th>
<th>$\mu_r$ (GHz)</th>
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<th>$\beta$, GHz</th>
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<tr>
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<td>50.000</td>
<td>15.000</td>
<td>0.957</td>
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</tr>
</tbody>
</table>

Lossy Magnetic Materials ($\varepsilon_r = 15.000$ - $j0.000$)

$\mu = \mu_0 - j\mu_r$, $\mu_0 (GHz) = \mu_0 (GHz)/f^\alpha$, $\mu_r (GHz) = \mu_r (GHz)/f^\beta$ (f in GHz)

<table>
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<tr>
<td>3</td>
<td>5.000, 0.974</td>
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</tr>
<tr>
<td>4</td>
<td>3.000, 1.000</td>
<td>15.000, 0.957</td>
</tr>
<tr>
<td>5</td>
<td>7.000, 1.000</td>
<td>12.000, 1.000</td>
</tr>
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</table>

Lossy Dielectric Materials ($\mu_r = 1.000$ - $j0.000$)

$\varepsilon = \varepsilon_0 - j\varepsilon_r$, $\varepsilon_0 (GHz) = \varepsilon_0 (GHz)/f^\alpha$, $\varepsilon_r (GHz) = \varepsilon_r (GHz)/f^\beta$ (f in GHz)

<table>
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<td>7</td>
<td>8.000, 0.778</td>
<td>10.000, 0.682</td>
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<tr>
<td>8</td>
<td>10.000, 0.778</td>
<td>6.000, 0.861</td>
</tr>
</tbody>
</table>

Relaxation-type Magnetic Materials ($\varepsilon_r = 15.000$ - $j0.000$)

$\mu = \mu_0 - j\mu_r$, $\mu_0 (GHz) = \mu_0 (GHz)/f^\alpha$, $\mu_r (GHz) = \mu_r (GHz)/f^\beta$ (f and $f_\alpha$ in GHz)

<table>
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<tr>
<td>16</td>
<td>25.000</td>
<td>3.500</td>
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</tbody>
</table>
3.2 Wide-Band Absorbing Coating for Planar Structures

The optimal wide-band absorbing layers for reducing reflection coefficient of the planar structures are addressed in this section. The reflection coefficient is determined using the equation

\[ \Gamma = \left| \frac{E'}{E} \right|. \]  

(3.3)

In this study, the reflection coefficient is minimized only for the normal incidence waves. The optimal RAM is obtained based on the available database summarized in Table 3.1. Although the materials are fictitious, they provide a good representative sample of a wide class of RAM coatings. The characteristics of these materials can be categorized in the following ways:

1) Lossless and frequency-independent dielectric materials (# 1-2),

2) Lossy magnetic (# 3-5) and lossy dielectric (# 6-8) frequency dependent materials,

3) Lossy magnetic frequency dependent materials (# 9-16) with the relaxation type characteristics.

As an example, the relative permittivity and permeability of materials #7 and #16 are plotted in Fig. 3.2. As observed, these materials have different characteristics as a function of frequency. Application of GA method allows one to obtain the best selection of the available RAM in the desired range of frequency.

The optimal planar coatings for reducing the reflection coefficient of the conducting planar structure, illuminated at normal incidence, are shown in Table 3.2. In this case, two optimization frequency ranges, namely, 0.2 – 2 GHz at 10 points and
Reflection coefficient of the coated structure is shown in Fig. 3.3 and it reveals a good agreement with the published data by Michielssen et al. [22]. As observed, the RAM coating decreases the reflection coefficient about $32\, dB$ for the first optimization range. For optimization range of $0.2-10\, GHz$, the reflection coefficient is almost flat and it is reduced by about $18\, dB$.

One may wonder what happens if the GA optimizer is allowed to use only a single material during the optimization process. The effect of using a single layer coating in reducing the reflection coefficient of the planar structure in the frequency range $0.2-2\, GHz$ is presented in Fig. 3.3. The thickness of the coating is allowed to change between $0.000 \leq T_i (cm) \leq 1.000$ in the GA procedure. The optimal RAM is the material #4 with thickness $T_i = 0.249\, cm$. It shows that based on these materials, using one coating only reduces the reflection coefficient about $15\, dB$ in the optimization frequency range.

The most difficult part in reducing the reflection coefficient of the planar structure occurs at very low frequency. In this range of frequency, the finite conductivity in most of the available materials makes the coating act like a perfect conductor and the energy cannot enter the coating to be absorbed [25]. At high frequencies around $10\, GHz$, the relative permittivity and permeability of the materials are decreased and they cannot control the reflection coefficient of the structure effectively. Using a more effective available database, and increasing number of layers and their thickness can further reduce the reflection coefficient.
Fig. 3.2: Relative permittivity and permeability of materials #7 and #16, from Table 3.1, as a function of frequency.

Table 3.2: Optimum RAM for the conducting plane, (a) \(0.2 \leq f_{\text{opt}}(GHz) \leq 2\), (b) \(0.2 \leq f_{\text{opt}}(GHz) \leq 10\).

<table>
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<th>Material #</th>
<th>(T) (cm)</th>
<th>Layer</th>
<th>Material #</th>
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</tbody>
</table>
3.3 RCS Reduction of Cylindrical and Spherical Structures

In this section, the modal solution/GA technique is applied to investigate the RCS reduction of the cylindrical and spherical structures for both monostatic and bistatic cases. The optimal RAM and some of the important considerations in monostatic and bistatic RCS reduction of these structures are addressed.

3.3.1 Monostatic RCS Reduction

To obtain the optimal composite coating for monostatic RCS reduction of cylindrical and spherical structures, the genetic algorithm optimizer is used to synthesize a wide-band
absorbing RAM in the frequency range of 0.2 – 2 GHz. In the cases studied here, the optimal RAM is determined for representative conducting structures with 148 cm diameter (electric size is changed from about 1λ to 10λ in the optimization range 0.2 ≤ f ≤ 2 GHz).

For a cylindrical structure, the RCS for TMz mode is obtained using the definition of 2-D RCS as

\[ \sigma_{2-D} = \lim_{\rho \to 0} \left[ 2\pi\rho \frac{|E'|^2}{|E|^2} \right]. \]  

(3.4)

RCS for TEz case is similarly determined. The corresponding coatings and their thickness are presented in Table 3.3(a). In the optimization procedure one could use TMz or TEz modes separately; however, from practical consideration, the optimum coating is determined for both polarizations simultaneously. Fig. 3.4 displays the RCS of the conducting cylinder, without and with coating for both TMz and TEz modes. It is observed that the optimal absorbing materials reduce the RCS of the conducting cylinder by about 27 dB in the optimization frequency range. In the resonance region, the optimal coating works very well as an absorber; and as one extends the solution to the Rayleigh region, it is found that this effect is reduced. At the low frequencies, most of the materials act as a perfect conductor, and the coating cannot absorb the energy of the incident wave as well as in the resonance region. Additionally, at the higher frequency range, the relative permittivity and permeability of the materials are reduced and the RCS cannot be reduced effectively.
Fig. 3.4 also shows the RCS of the coated cylindrical structure based on the Geometrical Optics (GO) formula as [26]

\[
\sigma_{2-D} = \left| \Gamma_{cylinder} \right|^2 \cdot \pi a_{coated} \quad (3.5)
\]

where \( \Gamma_{cylinder} \) is the reflection coefficient of the planar structure coated with the optimal cylindrical coating (Table 3.3(a)), and \( a_{coated} \) is the outer radius of the coated cylindrical structure. A good agreement between the modal solution and GO approximation is observed.

For the spherical target the RCS is obtained as

\[
\sigma_{3-D} = \lim_{r \to \infty} \left[ 4\pi^2 \left( \frac{|E'|^2}{|E|^2} \right) \right] \quad (3.6)
\]

The optimal RAM coating for reducing the RCS of the conducting sphere in the optimization frequency range of \( 0.2 \text{−} 2\text{GHz} \) is shown in Table 3.4(a). The RCS has been plotted in Fig. 3.5. As observed, the optimum coating reduces the RCS of the conducting sphere by about \( 31\text{dB} \) in the optimization frequency range. In the resonance region, the optimal coating is a very good absorber and as one extends into the Rayleigh region due to the finite conductivity nature of the materials considered in this study, the RCS of the structure cannot be reduced as well as in the resonance region. In addition, at the higher frequencies, as mentioned earlier, the RCS cannot be controlled effectively.

Fig. 3.5 also shows the RCS computation based on the geometrical optics formula as [26]

\[
\sigma_{3-D} = \left| \Gamma_{sphere} \right|^2 \cdot \pi a_{coated}^2 \quad (3.7)
\]
where $\Gamma_{\text{sphere}}$ is the reflection coefficient from the planar structure using the optimized spherical coating (Table 3.4(a)), and $a_{\text{coated}}$ is the outer radius of the coated spherical structure. It is observed that the computational results for the frequencies above 0.3$GHz$ closely resemble the results obtained using the GO formula.

As noted from Fig. 3.5, the RCS for the spherical structure is more similar to the RCS of the cylindrical structure for TE$_z$ mode (Fig. 3.4). The reason is that for TE$_z$ case the surface current, as shown in Fig. 3.6, is in the $\phi$ direction, and it shows similarity to the current direction on the spherical geometry. However, in TM$_z$ case the current is in the $z$ direction (Fig. 3.6), and does not resemble any current distribution on the spherical surface.

### 3.3.2 Bistatic RCS Reduction and the Deep Shadow Region

In the previous section, the optimal coatings for monostatic RCS reduction of cylindrical and spherical structures were successfully obtained. The goal of this section is to investigate the effect of the designed RAM’s for reducing RCS of these conducting structures in other directions.

Fig. 3.7 presents the bistatic RCS of the cylindrical structure with and without coating (Table 3.3(a)), for TE$_z$ case, in the different directions $\phi = 180^\circ$ (monostatic), $135^\circ$, $90^\circ$, $45^\circ$, and $0^\circ$ (forward scattering). As observed, the bistatic RCS in the lit region as well as the monostatic RCS is strongly reduced. Although the optimal coating is achieved for reducing the monostatic RCS, due the existence of lossy materials, this
coating also reduces the RCS in other directions except in the deep shadow region. This can be explained by recognizing the fact that in the deep shadow region the scattered field is usually 180° out of phase with the incident field and so there should be a high level of scattered field, no matter which type of composite coating is used. Note that at low frequencies, as mentioned earlier, the coating is almost the same as a perfect conductor and it only increases the size of the cylinder.

To further evaluate the possibility of RCS reduction in the deep shadow region, an optimization was performed to specifically reduce the bistatic RCS of the conducting cylinder in the deep shadow direction (\(\phi = 0^\circ\)). Table 3.3(b) shows the optimal materials for this trial. The bistatic RCS in the deep shadow region is plotted in Fig. 3.8 and as illustrated even this optimal coating cannot reduce the RCS in the deep shadow region. For TM\(_z\) case, similar observations for the bistatic RCS are obtained.

Bistatic RCS of the conducting sphere, coated with the optimal RAM for reducing the monostatic RCS (Table 3.4(a)), is explored in Fig. 3.9. The bistatic RCS is plotted in E-plane for different elevation angles \(\theta = 180^\circ\) (monostatic), 135°, 90°, 45°, and 0° (forward scattering). Due to the existence of lossy materials, the bistatic RCS in the lit region as well as the monostatic RCS is reduced considerably. However similar to the cylindrical structure, in the deep shadow region the scattered field is 180° out of phase with the incident wave and there is a high level of scattered field, as shown in Fig. 3.9(e). Table 3.4(b) and Fig. 3.10 show the optimal RAM and bistatic RCS of the spherical structure in the deep shadow region (\(\theta = 0^\circ\)), and as mentioned, the optimal coating only
increase the size of the sphere and cannot reduce the bistatic RCS in the deep shadow region. The bistatic RCS in the H-plane has almost the same behavior as the E-plane.

Table 3.3: Optimum RAM for the 148 cm diameter conducting cylinder \( 0.2 \leq f_{\text{opt}} (GHz) \leq 2 \).

(a) Monostatic \( (\phi = 180^\circ) \), (b) Bistatic \( (\phi = 0^\circ) \).

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Table 3.4: Optimum RAM for the 148 cm diameter conducting sphere \( 0.2 \leq f_{\text{opt}} (GHz) \leq 2 \).

(a) Monostatic \( (\theta = 180^\circ) \), (b) Bistatic \( (\theta = 0^\circ) \).

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Fig. 3.4: RCS of the conducting cylinder, without and with coating. RCS based on GO formula is also shown.

Fig. 3.5: RCS of the conducting sphere, without and with coating. RCS based on GO formula is also shown.
Fig. 3.6: Current distributions on the cylindrical structure for TM\textsubscript{z} and TE\textsubscript{z} modes, compared with the spherical structure. Note the surface current flow on the cylindrical structure for TE\textsubscript{z} mode has closer similarity to the nature of the current flow on the spherical structure.
Fig. 3.7: Bistatic RCS of the conducting cylinder (TE\textsubscript{z} case), without and with coating (Table 3.3(a)): (a) \( \phi = 180^\circ \) (monostatic), (b) \( \phi = 135^\circ \), (c) \( \phi = 90^\circ \), (d) \( \phi = 45^\circ \), (e) \( \phi = 0^\circ \) (forward scattering).

\[ \sigma_{\text{dB}} / \lambda (\text{GHz}) \]

- Without coating
- With coating

- Without coating
- With coating

- Without coating
- With coating

- Without coating
- With coating

- Without coating
- With coating
Fig. 3.8: Bistatic RCS of the conducting cylinder (TEz case) in the deep shadow region (\( \phi = 0^\circ \)), without and with coating (Table 3.3(b)). Optimal coating is determined for the forward scattering.
Fig. 3.9: Bistatic RCS of the conducting sphere (E-plane), without and with coating (Table 3.4(a)). (a) \( \theta = 180^\circ \) (monostatic), (b) \( \theta = 135^\circ \), (c) \( \theta = 90^\circ \), (d) \( \theta = 45^\circ \), (e) \( \theta = 0^\circ \) (forward scattering).

In order to show the range of RCS variations, the dynamic range in these plots varies from \(-60 \text{ dB} \) to \(70 \text{ dB} \) in contrast to the cylindrical case with an \(80 \text{ dB} \) dynamic range (Fig. 3.7).
3.4 GA with Hybrid Planar/Curved Surface Implementation

This section focuses on designing an optimal RAM coating for the curved surfaces using the GA technique via a hybrid planar/curved surface population initialization. In general, obtaining the optimal coating for the curved surfaces compared to the planar structures is more complex and much more time consuming. Clearly seeking an efficient optimization procedure for curved surfaces is desirable. For instance, the computer run time in obtaining the reflection coefficient of the coated planar structure on an HPC-180 workstation is about 0.0006 sec., while the RCS of the coated spherical structure is obtained in about 0.252 sec. This shows that there is a huge computational time...

Fig. 3.10: Bistatic RCS of the conducting sphere (E-plane) in the deep shadow region (θ = 0°), without and with coating (Table 3.4(b)). Optimal coating is determined for the forward scattering.
difference in synthesizing the wide-band absorbing materials for spherical structure compared to the planar structure. This section investigates the question of the effectiveness of using the best population for the planar RAM as the initial population in the GA technique in designing an optimal RAM for curved surfaces.

Fig. 3.11 shows the RCS of a conducting sphere coated with the optimal planar coating (Table 3.2(a)), and compares it with the RCS of a sphere coated with the optimum spherical coating (Table 3.4(a)). It is observed that the optimized coating corresponding to the planar structure creates an almost the same characteristic as with the optimal spherical coating and reduces the RCS of the spherical structure considerably. Note that the optimal planar coating on the HPC-180 workstation is achieved in about 1-min., while it takes approximately 7-hr. to obtain the optimal spherical coating on this workstation. This computational time is for evaluating the RCS for 100 generations (initial population was randomly selected), included 100 individuals per generation, at 10 different frequency points (i.e., $10^5$ RCS computations). The convergence curve for this optimization process is shown in Fig. 3.12. It is observed that it takes nearly 40 generations before the convergence curve approaches the final result.

Next, the GA optimizer is initiated using as its initial population the best population obtained from the optimal design of the planar structure to design the optimal spherical coating as represented in Table 3.5. The convergence curve for this implementation is also shown in Fig. 3.12. It is clearly observed that the optimization process requires significantly less number of generations to approach the final design. As expected due to the inherent robustness of the GA optimizer, no matter what the initial
population the final optimal result is almost independent of the initial population. However, starting with a better initial population one can expedite the convergence rate noticeably. Fig. 3.11 compares RCS of RAM coated sphere using various schemes as discussed above.

These observations may suggest that the optimal coating for an arbitrary shaped structure can be efficiently obtained utilizing the GA with planer/curved surface implementation. In the first step, a multi-layered planar structure is analyzed to obtain the best population for minimizing its reflection coefficient. Next, this population is used as the initial population for the GA optimizer applied to the curved surface to allow for rapid determination of the optimal coating for the curved structure.

Table 3.5: Optimum RAM designed by GA hybrid planar/curved surface implementation for the 148 cm diameter conducting sphere (0.2 ≤ f_\text{opt} (GHz) ≤ 2).

<table>
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<td>5</td>
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Fig. 3.11: Comparison of the RCS of the conducting sphere coated with the optimal spherical coating (Table 3.4(a)), optimal planar coating (Table 3.2(a)), and optimal spherical coating using GA hybrid planar/curved surface implementation (Table 3.5).

Fig. 3.12: Convergence curve of the GA hybrid planar/curved surface implementation compared to the GA method for RCS reduction of the conducting sphere.
3.5 Summary

In this chapter, a novel procedure for synthesizing RAM in a wide-band frequency range for reducing RCS of curved targets is presented. Multi-layered planar, cylindrical, and spherical canonical structures are analyzed using the vector wave solution of Maxwell’s equations. Subsequently, the genetic algorithm optimization technique is integrated with the modal solutions to obtain the optimal absorbing materials for RCS reduction. Since the genetic algorithm produces the global solution without requiring much information about the solution domain, it has been found to be a very effective method for obtaining the optimal coating.

An in-depth study was performed to evaluate the potential usage of the optimal planar coating as applied to the curved surfaces. It is observed that the optimal planar coating can noticeably reduce RCS of curved structures. This observation was essential in introducing a novel efficient GA implementation using as part of its initial generation the best population obtained for the planar RAM design. These results suggest that the optimal RAM for a surface with an arbitrary curvature may be efficiently determined by applying the GA technique.

It is shown that for the structures studied in this paper by a proper composition of absorbing materials, the monostatic RCS of the planar, cylindrical, and spherical conducting structures is reduced about $27 dB$ in the optimization frequency range of $0.2 – 2 GHz$. In the resonance region, the coating absorbs the energy of the incident wave strongly; however, as one extends to the Rayleigh region the finite conductivity of some of the RAM makes the coating act like a perfect conductor and the RCS cannot be
reduced as effectively as in the resonance region. Around the geometrical optics region, the relative permittivity and permeability of the materials are decreased and the coating cannot control the RCS of the structure effectively.

The bistatic RCS reduction of the coated conducting cylindrical and spherical structures has also been investigated. It has been observed that, although the optimal coating was achieved for reducing the monostatic RCS, due to the existence of loss, it has also reduced the bistatic RCS in the lit region. In the deep shadow region no matter which type of coating is used the scattered field is almost $180^\circ$ out of phase with the incident wave. Therefore, the optimal RAM only increases the size of the structure and cannot reduce the bistatic RCS.

In addition, the effectiveness of the optimal planar coating in designing the optimal RAM for the curved surfaces is investigated. It has been shown that the optimal planar coating has almost the same effect as the optimal spherical coating in reducing the monostatic RCS of spherical structures. Furthermore, the best population for the planar coating is integrated as the initial population for an efficient GA technique to obtain the optimal coating for the spherical structure. It has been shown that this implementation of GA has a very fast convergence rate. This suggests that for the sake of the computational time reduction, the optimal RAM for curved surfaces can be obtained using the GA technique with the hybrid planar/curved strategy.
Chapter 4

Design Optimization of the Non-Uniform Luneburg and 2-Shell Lens Antennas

Searching for applicable antenna concepts for multi-beam scanning at millimeter and microwave frequencies in mobile communication systems, remote sensing, and radio astronomical, has renewed interest in multi-shell spherical lenses [6], [27]-[30]. These types of lenses transform the feed’s spherical wave front into a plane wave [31]-[33]. Since the mathematical principle of the lens is based on geometrical optics, it can operate over a broad band of frequencies. On the other hand, spherical symmetry of the lens allows for multi-beam scanning by placing an array of feeds around the lens. These attractive features of the multi-shell spherical lenses were the reason for their applications as early as 1950 [34]-[37]. While both multi-shell spherical lenses and phased array antennas can be used for beam scanning, the latter has a narrower frequency bandwidth.

An ideal Luneburg lens antenna consists of a dielectric sphere with varying permittivity ranging from two at the center to one at the surface. In practice, the Luneburg lens is constructed as a radially uniform multi-shell spherical lens that efficiently focuses the feed energy. However, in order to achieve the typically demanded high performance antenna characteristics, a large number of shells may be required. By
decreasing the number of shells the directivity is decreased, and substantially higher levels of unwanted grating lobes are generated. This deficiency in the radiation performance of the lens can be improved by designing a non-uniform lens to overcome the grating lobe generations. The main question is how to choose the optimum layer widths and dielectric constants for the shells, while minimizing the number of shells necessary without introducing grating lobes.

This chapter focuses on the optimum synthesis of the non-uniform lens antennas based on the genetic algorithm, in order to obtain the optimal material and thickness of the spherical shells. Since it is desired to simultaneously control the achievable gain with reduced grating lobe envelope, an adaptive cost function is used in the process of GA optimization.

Various lens geometries including air gaps and feed offset from lens surface are successfully optimized utilizing the vector wave solution of Maxwell’s equations (Chapter 2) integrated with GA. Many useful engineering design guidelines have been suggested for the optimum construction of the lens. The results have been satisfactory and demonstrate the utility of the GA/adaptive-cost-function algorithm. Additionally, radiation characteristics of a novel 2-shell lens antenna have been studied and its performance is compared to the Luneburg lens. Furthermore, near field behaviors of the optimized lens antennas are presented.
4.1 Modal Solution Integrated with GA/Adaptive-Cost-Function

The challenge in this section is to properly integrate the modal solution of Maxwell’s equations for the multi-layered spherical structure with the GA/adaptive-cost-function in order to present the optimal material and thickness of the spherical shells of the lens antenna.

In the context of optimizing the radiation performance of the lens antenna with $M$ spherical shells, the objective or fitness function $F$ is defined by

$$F(e_{r_1}, T_1, e_{r_2}, T_2, \ldots, e_{r_M}, T_M) = \alpha \cdot G_0 + \beta \cdot \min(f(\theta) - G(\theta))_{\text{sidelobe region}}$$

(4.1)

where $e_{r_j}$ and $T_j$ represent the material choice for each shell and its thickness, respectively. $G(\theta)$ is the gain pattern of the lens antenna in E-plane (note that a lens antenna typically has similar E&H-plane patterns) and $G_0$ is its maximum value. The cost function $F$ attains the maximum gain with low sidelobe levels, for an optimum configuration. By a proper setting of parameters $\alpha, \beta$, and the sidelobe envelope function $f(\theta)$, one can effectively control the gain and sidelobe levels.

In order to apply the GA method to the lens structure, the material and thickness of each shell are represented in a binary code as

$$L_j = M_j T_j = [m_{j1}m_{j2} \ldots m_{jN_{\text{mat}}}][t_{j1}t_{j2} \ldots t_{jN_{\text{th}}}]$$

(4.2)

where $M_j$ and $T_j$ are the finite sequences of binary bits representing the material and thickness of $j^{th}$ spherical shell, respectively. The entire structure is subsequently represented by the sequence $G = L_1 L_2 \ldots L_M$. To obtain the optimal material and thickness
with 3-digit resolution, each of the sequences $M_j$ or $T_j$ is included with 10 binary bits $N_{mb} = N_{tb} = 10$. The size of the population is $N_{pop} = 100$. For each case, EM fields and the cost function $F$ are accurately computed. The evaluated cost function is suitably incorporated into the GA method to present the optimal layered spherical lens antenna. The optimization process has been depicted in Fig. 4.1. Note that in order to fix the outer diameter of the lens a proper constraint has been applied on the total thickness of the shells, as will discussed in Chapter 4.3.

![Fig. 4.1: Genetic algorithm integrated with modal solution of Maxwell’s equations for designing optimum non-uniform lens antenna.](image)
4.2 Uniform Luneburg Lens Antenna and its Shortcomings

A Luneburg lens antenna transforms the point source radiation into the plane wave and vice versa. An ideal Luneburg lens consists of a spherically symmetric dielectric sphere with continuous varying permittivity from two at the center of inner core to the one at the outer surface, i.e., \( \varepsilon_r = 2 - \left( \frac{r}{a} \right)^2 \), where \( a \) is the radius of the sphere. In practice, however, the Luneburg lens is constructed by a finite number of spherical shells, retaining reasonable performance. Table 4.1 shows the material and thickness for a 5-shell \( 30\lambda \) (\( \lambda \) in free space) diameter uniform lens. Usually feed horns or open-ended waveguides are used to illuminate the lens. In general, one could model the feed by its equivalent dipole aperture field. In this paper, a typical horn antenna is modeled using an end-fire antenna consisting of four infinitesimal dipoles, as shown in Figs. 4.2 and 4.3. The end-fire antenna has cross polarization isolation around \( 17 \) dB with about 7% spill-over efficiency.

For a Luneburg lens antenna with fixed diameter, the radiation performance is improved by increasing the number of shells, but the performance becomes saturated with \( \approx 1\lambda \) thickness for each shell. The radiation pattern of a 10-shell \( 30\lambda \) diameter lens is shown in Fig. 4.4. As observed the gain is about \( 37.73 \) dB (aperture efficiency compared to the uniform aperture distribution is \( \eta = 67\% \)). Note that due to the existence of the periodic-like uniform shells, there appear some grating lobes in the pattern occurring around \( \theta = 30^\circ \). By reducing the number of shells the thickness of each shell is increased and so the grating lobes are moved closer to the main beam, as shown for a 5-
shell lens (see Fig. 4.4). In this case the gain of the lens is decreased to 36.46\,dB (\(\eta = 50\%\)) and the grating lobes occur closer to the main beam, around \(\theta \approx 17^\circ\). Note that the location of the grating lobes for the periodic-like uniform Luneburg lens can be estimated from the simple formula \(\theta = \sin^{-1}(\lambda / T)\) (\(T\) is the thickness of the spherical shell), which gives \(\theta = 41.81^\circ\) and \(\theta = 19.47^\circ\) for the 10-shell and 5-shell lenses, respectively.

Position of the feed antenna is the other important aspect in designing the Luneburg lens antenna. From the geometrical optic point of view when the phase center of the feed is on the surface, the lens antenna focuses the feed’s energy at the infinity. However, in practice this condition may not be fulfilled and the phase correction of the lens antenna becomes worse. The radiation pattern of a 5-shell 30\(\lambda\) diameter lens when the feed is 0.5\(\lambda\) away the surface of the lens is shown in Fig. 4.4. One observes that the dislocation of the feed degrades the lens performance by broadening the beam and dropping the gain of the lens to about 35.31\,dB (\(\eta = 38\%\)).

Air-gaps between spherical shells of the Luneburg lens antenna may be unavoidable in the process of constructing the lens. This air gap affects the performance of the lens as illustrated in Fig. 4.4. The radiation pattern of the 5-shell 30\(\lambda\) diameter lens with 0.05\(\lambda\) air gap shows the effect of the air gap by decreasing the gain of the lens to about 35.46\,dB (\(\eta = 40\%\)).

From these plots, it is demonstrated that a 10-shell 30\(\lambda\) lens antenna has the superior performance. Although, a 5-shell lens antenna with less number of spherical
shells has the easier construction, however, due to the larger thickness of each shell the grating lobes occur closer to the main beam degrading the performance of the antenna. An optimized non-uniform 5-shell lens antenna is introduced in the next section to noticeably improve the radiation characteristics of the uniform lens including gain, side lobe level, and grating lobes.

Table 4.1: Design parameters of a 5-shell 30\(\lambda\) diameter uniform Luneburg lens antenna.

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Fig. 4.2: 5-shell 30\(\lambda\) diameter Luneburg lens antenna. An end-fire antenna consisting of four infinitesimal dipoles models the actual feed.
Fig. 4.3: Radiation pattern of the feed antenna. Feed has cross polarization isolation around 17 dB with about 7% spill-over efficiency.

Fig. 4.4: Gain pattern of the \(30\lambda\) diameter uniform Luneburg lens antenna in the \(x-z\) plane. Note the appearance of the unwanted grating lobes for 5-shell case.
4.3 Non-Uniform Luneburg Lens Antenna

This section focuses on the key design aspects of a non-uniform Luneburg lens antenna using the genetic algorithm. The goal is to determine the permittivity and thickness of each spherical shell for a 5-shell $30\lambda$ diameter lens antenna such that the maximum gain is achieved. However, besides maximizing the gain, the level of sidelobes should remain low. The challenge then is to investigate the tradeoffs between these conflicting objectives.

4.3.1 GA Implementation

In this study, the number of shells is $M = 5$, and $\varepsilon_j$ is allowed to change between 1.0 to 2.0. In the procedure of the optimization, the total diameter of the lens antenna is kept fixed at $30\lambda$. In other words, the optimum thickness of the spherical shells should satisfy the following equation

$$T_1 + T_2 + \cdots + T_5 = 15\lambda.$$  \hspace{1cm} (4.3)

For imposing this condition, the thickness of each shell can be allowed to change arbitrarily between 0 to $15\lambda$, and then the appropriate thickness is selected based on a scaling approach. This means that the appropriate thickness of each shell is obtained using the scaling coefficient $\zeta = \frac{D_{lens}}{D_{GA}}$. $D_{lens}$ is the actual diameter of the lens, and $D_{GA}$ is the determined diameter from the GA procedure. However, this method is time consuming.
As mentioned earlier, the ideal Luneburg lens has a dielectric constant 
\[ \varepsilon_r = 2 - \left( \frac{r}{a} \right)^2 \] with the slope 
\[ \nu = -2\frac{r}{a^2} \]. It reveals that, the inhomogeneous 
permittivity of the lens has the larger slope variation near the surface of the lens 
compared to the inside of the lens. Therefore, in order to design a high performance 
multi-shell lens antenna, the larger number of spherical shells has to be used near the 
surface of the lens, which may be explained as follow.

Each spherical shell can be considered as a focusing device where by choosing 
the optimal material the phase error is reduced and the waves are focused properly. The 
quality of focusing is inversely proportional to the projected area of the spherical shells, 
and to obtain a good design the inner shells should have a bigger thickness compared to 
the outer shells; and then a larger number of layers is necessary near the surface of the 
 lens compared to the inside of the lens. Therefore, to achieve a fast optimum design, the 
following constraint on the thickness of spherical shells is imposed:

\[ 0 \leq T_i \leq 2.9\lambda, \ 0 \leq T_2 \leq 2.9\lambda, \ 0 \leq T_3 \leq 5\lambda, \ 0 \leq T_4 \leq 4\lambda, \]  \hspace{1cm} (4.4) 

whereas the inner thickness, \( T_5 \), is obtained based on the Eq. (4.3). Imposing constraints 
such as Eq. (4.4) implicitly allows a bigger range for the inner shells in the GA process 
and fulfills the aforementioned requirements.

### 4.3.2 Non-Uniform Lens with Improved Gain

For achieving the maximum gain the parameters \( \alpha \) and \( \beta \) in Eq. (4.1) are chosen 1 and 
0, respectively. Table 4.2(a) and Fig. 4.5 show the design parameters and the gain pattern 
of a 5-shell non-uniform lens. As observed, a thicker core compared to the uniform
design is obtained which helps the increment in the gain of the lens antenna by 1 dB to $G_0 = 37.47 \text{ dB} \ (\eta = 63\%)$. Additionally, the grating lobe is decreased about 6.5 dB with respect to the uniform design.

4.3.3 High Gain/Low Sidelobe Non-Uniform Lens

Utilizing an appropriate cost function into the GA method one can design a non-uniform lens antenna with high gain and remarkably reduced grating lobes. In this manner, the parameter $\beta$ and envelope function $f(\theta)$ are properly incorporated into the Eq. (4.1) to control the sidelobes of the lens. As noted earlier, the $30\lambda$ 10-shell uniform Luneburg lens antenna has the acceptable monotonically decaying sidelobe variations. Therefore, the function $f$ is taken as the envelope of these sidelobes in the following form (as shown in Fig. 4.6)

$$f(\theta) = 12 - 38 \log(\theta^{1/5.8}) \ (dB). \quad (4.5)$$

In this case, a useful design is obtained by choosing $\alpha = 1$, and $\beta = 0.5$. The goal is to achieve the maximum gain while the sidelobes level is kept low, similar to a 10-shell lens. In order to create an adaptive cost function in the process of the GA optimization, the objective function $F$ is taken as the antenna gain (i.e., $\beta = 0$) in the first 20 generations. Next, the effect of both high gain and low sidelobes level are jointly used in the process of the optimization. This helps the optimization method to keep the antenna gain in a high level while decreasing the sidelobe levels.
The results are presented in Table 4.2(b) and Fig. 4.7. Note that the function $f$ was chosen to control the sidelobe levels in the angular range $2.5^\circ \leq \theta \leq 25^\circ$ at 50 points. To reduce the grating lobes, in general, the spherical shells should be very thin, which needs a large number of shells. However, in this study the lens antenna has only 5 layers and as indicated in Table 4.2(b), the GA optimizer makes the intermediate shell thicker to allow a smaller thickness for the neighboring layers. Fig. 4.7 shows that the gain is about $35.94\,dB$ ($\eta = 44\%$) and the grating lobes are decreased about $12\,dB$ with respect to the uniform design. Although, in this design the gain is dropped by $1.5\,dB$ compared to the previous case ($\alpha = 1, \beta = 0$), but the grating lobes are noticeably suppressed under the envelope function $f$.

The effectiveness of this adaptive optimization technique is shown in Fig. 4.8 by varying $\alpha$ and $\beta$ in Eq. (4.1). It is observed that simultaneous application of $\alpha$ and $\beta$ in the cost function provides the improved result. Fig. 4.9 shows a typical convergence curve of the GA method for three different sets of $\alpha$ and $\beta$. For $\alpha = 1$ and $\beta = 0$, the gain converges to the optimum value rapidly. However, for $\beta = 0$ the grating lobe cannot be controlled. When the optimization parameters $\alpha = 0$ and $\beta = 1$, there is no constraint on the directivity and the antenna gain is decreased significantly, as observed in Fig. 4.9. Therefore, $\alpha = 1$ and $\beta = 0.5$ values have provided an efficient adaptive optimization by controlling the gain and grating lobe of the lens antenna. Note that a Pareto GA’s [20] may be used to further refine the design by creating a set of optimized solutions.
4.3.4 Optimal Non-Uniform Lens with Feed Offset and Air Gap

The radiation pattern of a $30\lambda$ lens antenna when the phase center of the feed has been moved $0.5\lambda$ away from the surface, for two different optimization sets $\alpha = 1, \beta = 0$ and $\alpha = 1, \beta = 0.5$ has been plotted in Fig. 4.10. The design parameters of the lens have been shown in Table 4.3. It is observed that the pattern is improved with respect to the uniform design by correcting the phase center of the feed using the non-uniform lens design. In the first case, the gain is increased to about $36.73\, dB$ ($\eta = 53\%$) and in the second case, the grating lobe is also controlled using the adaptive cost function.

As mentioned earlier, the existence of the air gap between spherical shells may be unavoidable in the constructing process of the Luneburg lens antenna. The optimized material and thickness for a 5-shell lens with $0.05\lambda$ air gap, using cost function $F$ with two different optimization parameters $\alpha = 1, \beta = 0$ and $\alpha = 1, \beta = 0.5$ are presented in Table 4.4. Fig. 4.11 shows the radiation pattern of the optimized lens. The results show the validity of the GA method for designing such a lens antenna with air gap.

The other interesting aspect of this optimization technique is to design a practical lens antenna with $0.05\lambda$ air gap and feed $0.5\lambda$ away from its surface. The radiation pattern for the uniform Luneburg lens is shown in Fig. 4.12 and as observed the lens cannot focus the feed energy as well as before and the performance of the lens is decreased significantly. The characteristics of the lens are improved by designing a non-uniform lens as shown in Table 4.5, for two different optimization cases $\alpha = 1, \beta = 0$ and $\alpha = 1, \beta = 0.5$. The radiation pattern is plotted in Fig. 4.12. It is observed that the directivity of the optimum design for $\alpha = 1, \beta = 0$ is about $36.48\, dB$ ($\eta = 50\%$) and the
grating lobes are decreased about $5.2\, dB$ with respect to the uniform design. Using the adaptive cost function ($\alpha = 1, \beta = 0.5$) the grating lobes are better controlled as shown in Fig. 4.12.

Therefore, the radiation performance of the uniform Luneburg lens antenna is considerably improved by designing the non-uniform lens using the adaptive GA method. The antenna gain is increased and the grating lobes are suppressed noticeably. The sensitivity of the optimum design based on the tolerance in the dielectric constant $\varepsilon_r$ is also studied in this work. The radiation performance of the non-uniform optimized lens ($\alpha = 1, \beta = 0$), using 3-digit, 2-digit, and 1-digit accuracy in $\varepsilon_r$, is plotted in Fig. 4.13. It is observed that for a high performance lens design a dielectric constant with at least 2-digit accuracy should be used. Co-polarized and cross polarized fields of the optimized lens antenna are also presented in Fig. 4.14. It is noted that the non-uniform lens antenna has a cross polarization isolation of $23\, dB$. Clearly, lower cross polarization can be achieved using a more symmetric feed patterns.
Table 4.2: Design parameters of a 5-shell $30\lambda$ diameter non-uniform Luneburg lens antenna, 
(a) $\alpha = 1$, $\beta = 0$, (b) $\alpha = 1$, $\beta = 0.5$.

<table>
<thead>
<tr>
<th>Shell</th>
<th>$\varepsilon_r$</th>
<th>$T/\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.203</td>
<td>2.798</td>
</tr>
<tr>
<td>2</td>
<td>1.469</td>
<td>2.047</td>
</tr>
<tr>
<td>3</td>
<td>1.642</td>
<td>2.030</td>
</tr>
<tr>
<td>4</td>
<td>1.797</td>
<td>2.464</td>
</tr>
<tr>
<td>5</td>
<td>1.942</td>
<td>5.661</td>
</tr>
</tbody>
</table>

Table 4.3: Design parameters of a 5-shell $30\lambda$ diameter non-uniform Luneburg lens antenna. Feed is positioned $0.5\lambda$ away the surface of the lens, (a) $\alpha = 1$, $\beta = 0$, (b) $\alpha = 1$, $\beta = 0.5$.

<table>
<thead>
<tr>
<th>Shell</th>
<th>$\varepsilon_r$</th>
<th>$T/\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.325</td>
<td>1.870</td>
</tr>
<tr>
<td>2</td>
<td>1.190</td>
<td>1.478</td>
</tr>
<tr>
<td>3</td>
<td>1.556</td>
<td>2.917</td>
</tr>
<tr>
<td>4</td>
<td>1.739</td>
<td>2.689</td>
</tr>
<tr>
<td>5</td>
<td>1.899</td>
<td>6.046</td>
</tr>
</tbody>
</table>

Table 4.4: Design parameters of a 5-shell $30\lambda$ diameter non-uniform Luneburg lens antenna with $0.05\lambda$ air gap, (a) $\alpha = 1$, $\beta = 0$, (b) $\alpha = 1$, $\beta = 0.5$.

<table>
<thead>
<tr>
<th>Shell</th>
<th>$\varepsilon_r$</th>
<th>$T/\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.338</td>
<td>0.519</td>
</tr>
<tr>
<td>2</td>
<td>1.125</td>
<td>2.611</td>
</tr>
<tr>
<td>3</td>
<td>1.537</td>
<td>2.768</td>
</tr>
<tr>
<td>4</td>
<td>1.713</td>
<td>2.568</td>
</tr>
<tr>
<td>5</td>
<td>1.878</td>
<td>6.334</td>
</tr>
</tbody>
</table>

Table 4.5: Design parameters of a 5-shell $30\lambda$ diameter non-uniform Luneburg lens antenna with $0.05\lambda$ air gap. Feed is positioned $0.5\lambda$ away the surface of the lens, (a) $\alpha = 1$, $\beta = 0$, (b) $\alpha = 1$, $\beta = 0.5$.

<table>
<thead>
<tr>
<th>Shell</th>
<th>$\varepsilon_r$</th>
<th>$T/\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.080</td>
<td>0.915</td>
</tr>
<tr>
<td>2</td>
<td>1.195</td>
<td>2.450</td>
</tr>
<tr>
<td>3</td>
<td>1.599</td>
<td>2.895</td>
</tr>
<tr>
<td>4</td>
<td>1.736</td>
<td>2.394</td>
</tr>
<tr>
<td>5</td>
<td>1.883</td>
<td>6.146</td>
</tr>
</tbody>
</table>
Fig. 4.5: Gain pattern of the 5-shell $30\lambda$ diameter uniform and non-uniform optimized lens antenna in the $x-z$ plane. ($\alpha = 1, \beta = 0$)

Fig. 4.6: Gain pattern of the 10-shell $30\lambda$ diameter uniform Luneburg lens antenna with its sidelobe envelope function $f(\theta) = 12 - 38\log\left(\frac{\theta^2}{5.8^2}\right)(dB)$, in the $x-z$ plane.
Fig. 4.7: Comparison between 10-shell and 5-shell uniform design with the 5-shell non-uniform optimized lens in the \( x-z \) plane. (adaptive cost function: \( \alpha = 1, \beta = 0.5 \))

Fig. 4.8: Gain pattern of the 5-shell non-uniform optimized lens using different optimization parameters \( \alpha \) and \( \beta \).
Fig. 4.9: Convergence curve of the GA method for the 5-shell non-uniform optimized lens using different optimization parameters $\alpha$ and $\beta$.

Fig. 4.10: Gain pattern of the 5-shell $30\lambda$ diameter non-uniform optimized lens antenna in the $x-z$ plane. Feed is positioned $0.5\lambda$ away the surface of the lens. ($\alpha = 1, \beta = 0$ and $\alpha = 1, \beta = 0.5$)
Fig. 4.11: Gain pattern of the 5-shell $30\lambda$ diameter non-uniform optimized lens antenna with $0.05\lambda$ air gap in the $x-z$ plane. ($\alpha = 1, \beta = 0$ and $\alpha = 1, \beta = 0.5$)

Fig. 4.12: Gain pattern of the 5-shell $30\lambda$ diameter non-uniform optimized lens antenna with $0.05\lambda$ air gap in the $x-z$ plane. Feed is positioned $0.5\lambda$ away the surface of the lens. ($\alpha = 1, \beta = 0$ and $\alpha = 1, \beta = 0.5$)
Fig. 4.13: Gain pattern of the 5-shell $30\lambda$ diameter non-uniform optimized lens antenna using 3-digit, 2-digit, and 1-digit accuracy in the dielectric constant $\varepsilon_r$, in the $x-z$ plane. ($\alpha = 1, \beta = 0$)

Fig. 4.14: Co-polarization and cross polarization of the 5-shell $30\lambda$ diameter non-uniform optimized lens antenna in the $\phi = 0^\circ, \phi = 45^\circ$, and $\phi = 90^\circ$ planes. ($\alpha = 1, \beta = 0$)
4.4 2-Shell Lens Antenna

Geometrical optic theory predicts that a relatively high dielectric constant lens antenna can also collimate the feed energy in the front of the lens. A 2-shell lens antenna, proposed by T. L. Ap-Rhys [38], has the potential to satisfy the design requirements insofar as maximum gain and low side lobe level is concerned. This idea is supported by using the ray optic theory [31], [32], as illustrated in Fig. 4.15. Optical rays crossing the surface of the lens antenna at an angle $\theta$, can exit parallel to the axis of the lens by choosing suitable values for the dielectric constant of the shells. The core of the lens focuses the optical rays and the outer shell is a matching layer.

In this section, the GA technique is successfully applied to the design of 2-shell lens antenna, in order to obtain the optimal material and thickness of the core and matching layer. Optimal characteristics of the 2-shell $30\lambda$ diameter lens for two different optimization parameters $\alpha = 1, \beta = 0$ and $\alpha = 1, \beta = 0.5$ are obtained in Table 4.6, and as observed the matching layer is about a quarter wavelength. In this study, the dielectric constant $\varepsilon$ is allowed to change between 1.0-5.0, and the thickness of the outer shell is changed between 0 to $2\lambda$. The radiation pattern of the optimized 2-shell $30\lambda$ lens is plotted in Fig. 4.16. For optimization parameters $\alpha = 1, \beta = 0$, the directivity is about $37.48\, dB$ ($\eta = 63\%$) and additionally the envelope of sidelobes decays very well compared to the Luneburg lens antenna. However, the first sidelobe is increased, as observed from this plot. In this case, there are no grating lobes and then almost the same
result for the adaptive optimization with $\alpha = 1, \beta = 0.5$ is achieved, as shown in Fig. 4.16.

In order to show the effect of the matching layer in reducing the back-scattered waves, the radiation pattern of a homogeneous lens antenna with the proper dielectric constant $\varepsilon_r = 3.315$ (core without matching layer) is obtained in Fig. 4.16. As obtained, compared to the 2-shell lens antenna, the gain is dropped by about $0.24\,dB$ and the back-scattered waves are noticeably increased.

The manufacturing procedure of this type of lens is easier than for the Luneburg lens antenna. The inner core can be constructed from the Quartz-like material with dielectric constant $\varepsilon_{r_2} = 3.4$; and for the matching shell, Teflon with $\varepsilon_{r_1} = 2.1$ may be suitable. However, application of higher dielectric constant material could make these lenses heavier than the Luneburg lens.

The frequency dependence of the 2-shell lens antenna is studied in Table 4.7, and compared with the 5-shell non-uniform Luneburg lens. In each case, the same radiation characteristic for the feed antenna is used. As determined, the 2-shell and 5-shell lens antennas have the 63% efficiency at center frequency $f_0$ (lenses have been designed at this frequency). At 10% below the center frequency $f_0$, we obtain the same efficiency; however, the efficiency at 10% above the center frequency is decreased 4% and 5% for the 2-shell and 5-shell lens antennas, respectively. These lenses have lower efficiency at $0.7f_0$ and $1.3f_0$ (±30% coverage of the frequency bandwidth), as shown in Table 4.7.
The larger frequency bandwidth could be expected for the lens antenna with more number of spherical shells.

Fig. 4.15: Geometrical optic theory applied to a relatively high dielectric 2-shell lens antenna.

Table 4.6: Design parameters of a 2-shell lens antenna, (a) \( \alpha = 1, \beta = 0 \), (b) \( \alpha = 1, \beta = 0.5 \).

<table>
<thead>
<tr>
<th>Shell</th>
<th>( \varepsilon_r )</th>
<th>( T/\lambda )</th>
<th>Shell</th>
<th>( \varepsilon_r )</th>
<th>( T/\lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.224</td>
<td>0.218</td>
<td>1</td>
<td>1.868</td>
<td>0.209</td>
</tr>
<tr>
<td>2</td>
<td>3.315</td>
<td>14.782</td>
<td>2</td>
<td>3.287</td>
<td>14.791</td>
</tr>
</tbody>
</table>

Table 4.7: Frequency dependency of the 2-shell and 5-shell optimized lens antennas. (\( \alpha = 1, \beta = 0 \)).

<table>
<thead>
<tr>
<th>Frequency</th>
<th>( 0.7 f_0 )</th>
<th>( 0.9 f_0 )</th>
<th>( f_0 )</th>
<th>( 1.1 f_0 )</th>
<th>( 1.3 f_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficiency of 2-shell lens</td>
<td>53%</td>
<td>63%</td>
<td>63%</td>
<td>59%</td>
<td>46%</td>
</tr>
<tr>
<td>Efficiency of 5-shell lens</td>
<td>45%</td>
<td>63%</td>
<td>63%</td>
<td>58%</td>
<td>45%</td>
</tr>
</tbody>
</table>
Fig. 4.16: Gain patterns of the $30\lambda$ diameter optimized 2-shell and homogeneous ($\varepsilon_r = 3.315$) lens antennas in the $x-z$ plane. ($\alpha = 1$, $\beta = 0$ and $\alpha = 1$, $\beta = 0.5$)

4.5 Near Field Characteristics of the Lens Antenna

The focusing behavior of the lens antenna can be clearly illustrated by investigating the near field pattern of the lens. However, the near field computation of the $30\lambda$ diameter spherical lens antenna is time-consuming. To reduce the computational effort necessary to calculate the near field of the spherical lens, it is often sufficient to utilize a cylindrical lens instead of the spherical lens.

In the cylindrical structure, the electromagnetic fields can be represented in both TM$_z$ and TE$_z$ polarizations. In this study, the TE$_z$ polarization is considered where the component of the electric field parallel to the axis of the cylinder is equal to zero (E-
plane). This is consistent with the electric field in $x-z$ plane for the spherical lens antenna.

To validate the possibility of investigating the performance of the spherical lens utilizing the cylindrical lens, the radiation pattern of a 5-shell $30\lambda$ diameter spherical lens with the material and thickness determined from the optimization of a cylindrical structure (Table 4.8) is presented in Fig. 4.17. As observed the gain is about $36.21\, dB$ demonstrating that the optimal layers have almost the similar effects in both cylindrical and spherical structures for focusing the feed energy. This may suggest that the near field phenomenon of the spherical lens can also be effectively visualized using a cylindrical structure.

### 4.5.1 Focusing Behavior of the Luneburg Lens

Fig. 4.18 shows the computed near field pattern of the 5-shell $30\lambda$ diameter cylindrical Luneburg lens antenna using the same material and thickness for the optimized spherical lens for maximum gain (Table 4.2(a)). Note that the input radiated power is normalized to $1.0W$. As illustrated, the optimal lens antenna successfully collimates the rays from the feed and generates an almost plane wave on the exiting aperture.

### 4.5.2 Focusing Behaviors of the Constant and 2-Shell Lenses

Near field characteristics of the $30\lambda$ constant cylindrical lens antenna with dielectric constant $\varepsilon_r = 3.315$ is shown in Fig. 4.19. It is observed that the most part of the feed energy is drawn into the lens antenna. Additionally, some of the rays penetrate inside the
lens close to the critical angle, and they are almost totally reflected from the surface of
the lens. Therefore, many paths through the lens are observed where they add
constructively. Due to this effect, a glory region [39] with thickness about $6\lambda$ is
observed in the lens antenna.

As mentioned earlier, the performance of the constant lens antenna can be further
improved by designing a 2-shell lens antenna (Table 4.6(a)). Fig. 4.20 shows the near
field performance of the 2-shell cylindrical lens antenna. As illustrated in this figure, the
outer shell or matching layer reduces the back-scattered waves and the lens antenna
focuses the source radiation strongly. Note that the thickness of the matching layer is
$T = 0.218\lambda$ where again due to the totally reflected waves a glory region with thickness
about $6\lambda$ is produced. The electric field in this orbiting area is very strong representing a
hot glory region. It means that in this case there are more in-phase rays in the glory
region with respect to the constant lens antenna.

The aperture field of the 2-shell lens antenna is more uniform (i.e. less tapered)
compared to the 5-shell non-uniform lens resulting in a higher sidelobe level. These near
field characteristics may be used to better understand the radiation performance of the
lens and seek for a better design.
Table 4.8: Design parameters of a 5-shell $30\lambda$ diameter non-uniform cylindrical Luneburg lens antenna. ($\alpha = 1, \beta = 0$)

<table>
<thead>
<tr>
<th>Shell</th>
<th>$\varepsilon_r$</th>
<th>$T/\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.217</td>
<td>3.451</td>
</tr>
<tr>
<td>2</td>
<td>1.476</td>
<td>2.129</td>
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<tr>
<td>3</td>
<td>1.670</td>
<td>2.523</td>
</tr>
<tr>
<td>4</td>
<td>1.824</td>
<td>2.577</td>
</tr>
<tr>
<td>5</td>
<td>1.933</td>
<td>4.320</td>
</tr>
</tbody>
</table>

Fig. 4.17: Gain pattern of the 5-shell $30\lambda$ diameter non-uniform spherical lens antenna in the $x-z$ plane using the optimal material and thickness determined from the optimization of the cylindrical lens (Table 4.8). ($\alpha = 1, \beta = 0$)
Fig. 4.18: Near field contour plot of the 5-shell $30 \lambda$ diameter non-uniform cylindrical Luneburg lens antenna. (material and thickness are the same as the optimized 5-shell spherical lens (Table 4.2(a)))

Fig. 4.19: Near field contour plot of the $30 \lambda$ diameter constant cylindrical lens antenna. (material and thickness are the same as the constant spherical lens, ($\varepsilon_r = 3.315$))
4.6 Summary

In this work, non-uniform spherical lens antennas are synthesized using genetic algorithms. The multi-shell lens antenna is analyzed using the vector wave solution of Maxwell’s equations. The GA/adaptive-cost-function is applied to obtain the optimal material and thickness of the spherical shells for designing the optimal lens antenna with high performance radiation characteristics.

It is demonstrated that the radiation performance of the uniform Luneburg lens antenna with low number of shells is noticeably improved by designing a non-uniform lens; shells of varying widths and dielectric constants. By choosing an appropriate cost function, the gain of the 5-shell $30\lambda$ lens is increased about $1\, dB$, and also its grating
lobes decreased by $6.5\, dB$ with respect to the uniform design. Using the GA optimizer with adaptive cost function one can effectively control both directivity and sidelobe of the lens, as illustrated in a non-uniform lens design with improved gain and remarkably suppressed grating lobes. Additionally, even when the feed is not on the surface or an air gap exists between the shells, the GA method can optimize the lens parameters successfully.

Sensitivity of the optimized lens antenna based on the accurate knowledge of the dielectric constant $\varepsilon_r$ is also studied in this chapter. It is observed that for a high performance lens design, a dielectric constant with at least 2-digit accuracy should be used. Furthermore, cross polarization of the non-uniform lens antenna is computed and is shown that in this case there is about $23\, dB$ isolation. This isolation level can be further improved by using a feed with more symmetric patterns.

Design optimization of a novel 2-shell lens antenna consisting of a core coated with a matching layer is also presented. It is obtained that the lens has the potential to present a radiation performance with high gain and low sidelobe level variations. Also, it is shown that the 2-shell and 5-shell lens antennas have almost the similar bandwidth characteristics for the design cases studied in this paper.

The near field characteristics of the Luneburg, constant, and 2-shell lens antennas are also investigated. It is illustrated that the optimal lens antennas properly transfer the spherical feed radiation to the plane wave. The near field behavior of the lens can be used to better understand the performance of the antenna resulting in design of lens antenna with higher radiation characteristics.
Although this chapter focuses on the optimization over a continuous parameter set, however, as obtained in the previous chapter, the GA method is also capable to apply on the discrete structural parameters. This means that by using the optimization technique, the best material and thickness for the lens can be picked up among a finite set of materials in a database. In summary, the methodologies developed in this work will allow efficient design of multi-layered lens antennas based on the specifications of the desired application.
Chapter 5

Broadband Characterization of Complex EBG Structures

Periodic structures are abundant in nature and they have fascinated artists and scientists alike. When they interact with electromagnetic waves amazing features result. In particular, characteristics such as frequency stop-bands, pass-bands and band-gaps could be identified. Surveying the literature, one observes that various terminology have been used depending on the domain of the applications. These applications are seen in filter designs, gratings, Frequency Selective Surfaces (FSS), Photonic Band-Gaps (PBG) materials, etc [10]-[12]. In this dissertation these structures are classified under the broad terminology of “Electromagnetic Band-Gaps (EBG)” [7]. Generally speaking, EBG structures are 3-D periodic objects that prevent the propagation of the electromagnetic waves in a specified band of frequency for all angles of arrival and for all polarization states. However, in practice, it is very hard to obtain such complete band-gap structures and partial band-gaps are achieved. For example, filters typically cover the scalar situation and single angle of arrival. FSS typically cover limited angles of arrival and respond differently to polarization states. PBG typically cover in-plane angles of arrival and also sensitive to polarization
states. FSS terminology has been widely used in the microwave community while PBG terminology has been widely applied in the optical community.

Periodic structures are often analyzed using frequency domain integral equations and Method of Moment (MoM) solutions [10], [40]. Because of the required evaluation of periodic Green’s function, and the frequent desire to repeat the analysis at multiple frequencies, these approaches can become computationally intensive. Finite Difference Time Domain (FDTD) technique is an alternative approach to efficiently present the broadband characteristics of the periodic structures.

The main objective of this chapter is to develop an efficient and powerful computational engine utilizing FDTD technique to characterize and understand the unique propagation performances of the periodic EBG structures composed of complex scatterers of dielectrics and conductors of arbitrary shapes. There have been several methods proposed to implement the periodic boundary conditions in FDTD [9]. The sin/cos and split-field approaches are the most capable techniques in analyzing periodic structures. Basic mathematical concepts of these methods are developed in [9]. The focus in this chapter is to derive and organize the formulations and detail some of the steps, which one needs to effectively apply the FDTD technique.

5.1 Analysis Technique

In this section, the development of the FDTD technique with Periodic Boundary Condition/Perfectly Matched Layer (PBC/PML) in analyzing periodic structures at oblique incidence is presented. Taking advantage of the broadband analysis of the FDTD
approach provides great efficiency and accuracy when the structure is characterized to
demonstrate its frequency response. The Prony’s extrapolation scheme is also
incorporated to increase the efficiency of the computational engine. The main parts of
this computational engine as illustrated in Fig. 5.1 are discussed in the following sections.
Fig. 5.1: Powerful and efficient computational engine utilizing FDTD technique in the characterization of complex periodic EBG structures.
5.1.1 Yee Algorithm

To apply the FDTD algorithm to the simulation of the structure of interest, the medium is filled with the Yee unit cells (Fig. 5.1) in an arrangement, which ensures the continuity of the tangential fields across cell interfaces [8]. The update equations are used to compute the field values for each cell. The “leap-frog” nature between the electric and magnetic fields allows the equations in a time stepping manner to track the time evolution of the electromagnetic fields over the spatial grid.

5.1.2 Total Field/Scattered Field Formulation

The total field/scattered field formulation [8] is an attempt to realize the scattering of the plane wave illumination using the FDTD technique. In this manner, the FDTD region is divided into two distinct regions separated by a non-physical virtual surface, which serves the connecting boundary conditions, as shown in Fig. 5.1. Region 1, the inner zone of the lattice, is called the total field region and the Yee algorithm operates on the total field vector components. The interacting structure is embedded within this region. Region 2, the outer zone, is the scattered field region and the Yee algorithm operates on the scattered field vector components. The boundary conditions between the total fields (inner zone) and the scattered fields (outer zone) are imposed using the values of incident fields along the virtual connecting surface.

5.1.3 Periodic Boundary Conditions

Illuminating periodic structures at oblique incidence generates a cell-to-cell phase
variation between corresponding points in different unit cells, which causes the time
domain implementation to become challengeable. In this work, to achieve a powerful
computational tool, the Floquet type phase shift periodic boundary condition is
incorporated based on two recently introduced approaches as: (a) doubly excited
sine/cosine plane wave method (sin/cos), and (b) field transformation split-field
technique.

A. Sin/Cos Method

In the doubly excited sin/cos method [41], the instantaneous responses of the system to
the sine and cosine plane waves at the known boundary ends are combined to form a
complex phasor form. Next, the phase shifts in the frequency domain periodic boundary
condition are used to compute the complex fields at the unknown boundary ends. The
imaginary and real parts are used to truncate the sine and cosine grids, respectively.
These procedures for a periodic structure in the $y$ direction, as shown in Fig. 5.2, are
formulated as

![Diagram of periodic unit cell in the y direction for the sin/cos method.](image)

Fig. 5.2: Periodic unit cell in the $y$ direction for the sin/cos method.
where $k_y^i$ is the incident propagation vector in the $y$ direction. Notice that the sign of the propagation vector $k_y^i$ in Eqs. (5.1) and (5.2) is determined based on the phase shift of the points $A$, $C$ with respect to the points $B$, $D$. The above formulations can be simply extended for a periodic structure in both $y$ and $z$ directions. This approach is successfully employed to model various periodic structures. The stability of the sin/cos method is the same as the usual FDTD algorithm. The major drawback is that only a single frequency is calculated per simulation ($k_y^i$ in Eqs. (5.1) and (5.2) depends on the frequency) and is not efficient.

**B. Split-Field Technique**

In the split-field technique [9], [42], the field transformation is used to remove the time gradient across the grids. The derived Floquet-transformed Maxwell’s equations are discretized based on the field-splitting approach. The formulations and stability relation of the system are presented in the following sub-sections.
**B.1 Three-Dimensional Split-Field Formulations**

Fig. 5.3 depicts the scattering of an arbitrary incident plane wave by a periodic structure in the $y$ and $z$ directions. The structure is terminated to the PBC/PML walls. For an arbitrary periodic structure containing linear inhomogeneous anisotropic materials, Maxwell’s equations are represented as

![Diagram of scattering](image)

**Fig. 5.3**: Scattering of the plane wave by a periodic structure in the $y$ and $z$ directions: (a) Propagation direction and polarization of the incident wave, (b) Geometry of the structure, (c) Unit cell of the structure terminated to the PBC/PML walls.
\[ \nabla \times \mathbf{H} = j \omega \varepsilon_0 \vec{s}_e \mathbf{E} \]  \hspace{1cm} (5.3) \\
\[- \nabla \times \mathbf{E} = j \omega \mu_0 \vec{s}_m \mathbf{H} \]  \hspace{1cm} (5.4) \\

where \( \vec{s}_e = \vec{\varepsilon}_e + \vec{\sigma}_e / j \omega \varepsilon_0 \) and \( \vec{s}_m = \vec{\mu}_m + \vec{\sigma}_m^* / j \omega \mu_0 \). The parameters \( \vec{\varepsilon}_e, \vec{\sigma}_e, \vec{\mu}_m, \) and \( \vec{\sigma}_m^* \) represent the electric and magnetic properties of the material.

\[
\vec{\varepsilon}_e = \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}, \quad \vec{\sigma} = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{bmatrix} \]  \hspace{1cm} (5.5) \\
\[
\vec{\mu}_m = \begin{bmatrix} \mu_{rx} & 0 & 0 \\ 0 & \mu_{ry} & 0 \\ 0 & 0 & \mu_{rz} \end{bmatrix}, \quad \vec{\sigma}_m^* = \begin{bmatrix} \sigma^*_x & 0 & 0 \\ 0 & \sigma^*_y & 0 \\ 0 & 0 & \sigma^*_z \end{bmatrix}. \]  \hspace{1cm} (5.6)

To remove the phase shift that is accumulated across the grid, a new set of field variables \( \mathbf{P} \) and \( \mathbf{Q} \) is introduced

\[
\mathbf{P} = \mathbf{E} e^{j (k_y^i y + k_z^i z)} 
\]  \hspace{1cm} (5.7) \\
\[
\mathbf{Q} = \eta_0 \mathbf{H} e^{j (k_y^i y + k_z^i z)} 
\]  \hspace{1cm} (5.8) \\

where \( k_y^i \) and \( k_z^i \) are the incident propagation vectors in the \( y \) and \( z \) directions, respectively. The free space wave impedance \( \eta_0 \) is used to normalize the equations. The periodic boundary conditions for the transformed field variables are then simply

\[
\mathbf{P}(x, l_y, z) = \mathbf{P}(x, 0, z) \]  \hspace{1cm} (5.9a) \\
\[
\mathbf{P}(x, y, l_z) = \mathbf{P}(x, y, 0) \]  \hspace{1cm} (5.9b) \\
\[
\mathbf{Q}(x, l_y, z) = \mathbf{Q}(x, 0, z) \]  \hspace{1cm} (5.10a) \\
\[
\mathbf{Q}(x, y, l_z) = \mathbf{Q}(x, y, 0) \]  \hspace{1cm} (5.10b)
The Maxwell’s equations (5.3) and (5.4) for the transformed field variables $P$ and $Q$ in the time domain are now given by ($j\omega \leftrightarrow \partial/\partial t$)

\[
\frac{\varepsilon_{rx}}{c} \frac{\partial P_x}{\partial t} + \sigma_x \eta_0 P_x = \frac{\partial Q_y}{\partial y} - \frac{k_y}{c} \frac{\partial Q_z}{\partial t} + \frac{k_z}{c} \frac{\partial Q_y}{\partial t},
\]

(5.11a)

\[
\frac{\varepsilon_{rx}}{c} \frac{\partial P_y}{\partial t} + \sigma_y \eta_0 P_y = -\frac{\partial Q_z}{\partial z} + \frac{k_z}{c} \frac{\partial Q_x}{\partial t} - \frac{k_x}{c} \frac{\partial Q_y}{\partial t},
\]

(5.11b)

\[
\frac{\varepsilon_{rx}}{c} \frac{\partial P_z}{\partial t} + \sigma_z \eta_0 P_z = -\frac{\partial Q_x}{\partial x} + \frac{k_x}{c} \frac{\partial Q_y}{\partial t} + \frac{k_y}{c} \frac{\partial Q_x}{\partial t},
\]

(5.11c)

\[
\frac{\mu_{rx}}{c} \frac{\partial Q_x}{\partial t} + \sigma_x^* \eta_0 Q_x = -\frac{\partial P_y}{\partial y} + \frac{k_y}{c} \frac{\partial P_z}{\partial t} - \frac{k_z}{c} \frac{\partial P_y}{\partial t},
\]

(5.12a)

\[
\frac{\mu_{ry}}{c} \frac{\partial Q_y}{\partial t} + \sigma_y^* \eta_0 Q_y = -\frac{\partial P_z}{\partial z} + \frac{k_z}{c} \frac{\partial P_x}{\partial t} + \frac{k_x}{c} \frac{\partial P_z}{\partial t},
\]

(5.12b)

\[
\frac{\mu_{rz}}{c} \frac{\partial Q_z}{\partial t} + \sigma_z^* \eta_0 Q_z = -\frac{\partial P_x}{\partial x} + \frac{k_x}{c} \frac{\partial P_y}{\partial t} - \frac{k_y}{c} \frac{\partial P_x}{\partial t},
\]

(5.12c)

where

\[
\hat{k}^i = \hat{x} \sin \theta^i \cos \phi^i + \hat{y} \sin \theta^i \sin \phi^i + \hat{z} \cos \theta^i
\]

(5.13a)

\[
\vec{k}^i = \hat{x} \cdot \hat{k}^i, \quad \vec{k}^i = \hat{y} \cdot \hat{k}^i, \quad \vec{k}^i = \hat{z} \cdot \hat{k}^i.
\]

(5.13b)

There are two important tasks compared to the normal FDTD update equations that has to be noted:

1- The presence of the space and time derivatives of the same variable in Eqs. (5.11) and (5.12) which needs the modification of the usual Yee leapfrog updating equations,
2- Modification of the Courant stability criterion because of the extra terms and factor multiplying the time derivatives.

The extra time derivatives on the right-hand sides of the Eqs. (5.11) and (5.12) can be eliminated by defining new variables $P_{ua}$ and $Q_{ua}$ ($u = x, y, z$) to split the EM fields in the following form

$$P_x = P_{xa} + \frac{\vec{k}_z^i}{\varepsilon_{rx}} Q_y - \frac{\vec{k}_y^i}{\varepsilon_{rx}} Q_z,$$
$$Q_x = Q_{xa} - \frac{\vec{k}_z^i}{\mu_{rx}} P_y + \frac{\vec{k}_y^i}{\mu_{rx}} P_z \tag{5.14a}$$

$$P_y = P_{ya} - \frac{\vec{k}_z^i}{\varepsilon_{ry}} Q_x,$$
$$Q_y = Q_{ya} + \frac{\vec{k}_z^i}{\mu_{ry}} P_x \tag{5.14b}$$

$$P_z = P_{za} + \frac{\vec{k}_y^i}{\varepsilon_{rz}} Q_x,$$
$$Q_z = Q_{za} - \frac{\vec{k}_y^i}{\mu_{rz}} P_x \tag{5.14c}$$

Substituting these variables into the systems (5.11) and (5.12) results in the equations that are discretized. The following six equations are updates for the $P_{ua}$ and $Q_{ua}$

$$\frac{\varepsilon_{rx}}{c} \frac{\partial P_{xa}}{\partial t} + \sigma_y \eta_0 P_{xa} = \frac{\partial Q_y}{\partial y} - \frac{\partial Q_z}{\partial z} + \frac{\sigma_y}{\varepsilon_{rx}} Q_z - \frac{\sigma_y}{\varepsilon_{rx}} \vec{k}_z^i - Q_y \tag{5.15a}$$

$$\frac{\varepsilon_{ry}}{c} \frac{\partial P_{ya}}{\partial t} + \sigma_z \eta_0 P_{ya} = -\frac{\partial Q_z}{\partial z} + \frac{\partial Q_x}{\partial x} + \frac{\sigma_z}{\varepsilon_{ry}} Q_x \tag{5.15b}$$

$$\frac{\varepsilon_{rz}}{c} \frac{\partial P_{za}}{\partial t} + \sigma_z \eta_0 P_{za} = -\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + \frac{\sigma_z}{\varepsilon_{rz}} Q_x \tag{5.15c}$$

$$\frac{\mu_{rx}}{\eta_0} \frac{\partial Q_{xa}}{\partial t} + \frac{\sigma_x^*}{\eta_0} Q_{xa} = -\frac{\partial P_y}{\partial y} + \frac{\partial P_z}{\partial z} - \frac{\sigma_x^*}{\eta_0 \mu_{rx}} P_z + \frac{\sigma_x^*}{\eta_0 \mu_{rx}} P_y \tag{5.16a}$$

$$\frac{\mu_{ry}}{\eta_0} \frac{\partial Q_{ya}}{\partial t} + \frac{\sigma_y^*}{\eta_0} Q_{ya} = -\frac{\partial P_x}{\partial x} - \frac{\partial P_z}{\partial z} - \frac{\sigma_y^*}{\eta_0 \mu_{ry}} P_z \tag{5.16b}$$
\[
\frac{\mu_{rz}}{c} \frac{\partial Q_{za}}{\partial t} + \frac{\sigma_z^+}{\eta_0} Q_{za} = -\frac{\partial P_x}{\partial x} + \frac{\partial P_y}{\partial y} + \frac{\sigma_z^+ \bar{k}_y^i}{\eta_0 \mu_{rz}} P_x. \quad (5.16c)
\]

Notice that there are some typographical mistakes in the update equations \( P_{za} \) and \( Q_{za} \) determined in [9].

Once the “a” portions of the split-fields (\( P_{za} \) and \( Q_{za} \)) are updated, the new values for the \( P_x \) and \( Q_x \) (normal components to the PML) are determined from Eqs. (5.14)

\[
\begin{aligned}
\left(1 - \frac{\bar{k}_y^i}{\mu_{rz} e_{rx}} - \frac{\bar{k}_z^i}{\mu_{ry} e_{rx}}\right) P_x &= P_{za} + \frac{\bar{k}_z^i}{\mu_{rx} e_{rx}} Q_y - \frac{\bar{k}_y^i}{\mu_{rx} e_{rx}} Q_x \\
\left(1 - \frac{\bar{k}_y^i}{\mu_{rz} e_{ry}} - \frac{\bar{k}_z^i}{\mu_{rx} e_{ry}}\right) Q_x &= Q_{za} - \frac{\bar{k}_z^i}{\mu_{rx} e_{ry}} P_y + \frac{\bar{k}_y^i}{\mu_{rx} e_{ry}} P_x.
\end{aligned}
\quad (5.17) \quad (5.18)
\]

Finally, the remaining fields (tangential components to the PML) are obtained using

\[
\begin{aligned}
P_y &= P_{ya} - \frac{\bar{k}_z^i}{e_{ry}} Q_x \\
P_z &= P_{za} + \frac{\bar{k}_y^i}{e_{rz}} Q_x \\
Q_y &= Q_{ya} + \frac{\bar{k}_z^i}{\mu_{ry}} P_x \\
Q_z &= Q_{za} - \frac{\bar{k}_y^i}{\mu_{rz}} P_x.
\end{aligned}
\quad (5.19a) \quad (5.19b) \quad (5.20a) \quad (5.20b)
\]

Each of the equations is spatially discretized with the usual Yee algorithm. However, to obtain the results with second order accuracy, spatial and temporal averaging for some of
the terms is needed. For instance, in Eq. (5.15a), temporal averaging for the nonderivative
term $P_{ua}$ in the left-hand side, and spatial averaging for the nonderivative terms $Q_z$ and $Q_y$
in the right-hand side are required. Furthermore, a dual time grid is introduced such
that the field components are computed at each half time step. The discretization of one
equation from each group of the update equations (“a” portion of the fields and normal
and tangential components to the PML) is obtained in the following

$$
P_{xa}^{n+1}_{i,j,k} = \left( \frac{-\xi_x(1-2\beta)}{1+\beta \xi_x} \right) P_{xa}^{n+1/2}_{i,j,k} + \left( \frac{1-\beta \xi_x}{1+\beta \xi_x} \right) P_{xa}^{n}_{i,j,k} + \\
\left( \frac{T_y}{\varepsilon_y (1+\beta \xi_x)} \right) \left( Q_z^{n+1/2}_{i,j+1,k} - Q_z^{n+1/2}_{i,j,k} \right) + \left( \frac{-T_z}{\varepsilon_x (1+\beta \xi_x)} \right) \left( Q_y^{n+1/2}_{i,j,k+1} - Q_y^{n+1/2}_{i,j,k} \right) + \\
\left( \frac{-\bar{k}_y^i \xi_x}{2\varepsilon_y (1+\beta \xi_x)} \right) \left( Q_z^{n+1/2}_{i,j+1,k} + Q_z^{n+1/2}_{i,j,k} \right) + \left( \frac{-\bar{k}_z^i \xi_x}{2\varepsilon_x (1+\beta \xi_x)} \right) \left( Q_y^{n+1/2}_{i,j,k+1} + Q_y^{n+1/2}_{i,j,k} \right) \\
(5.21)
$$

$$
P_{x}^{n+1}_{i,j,k} = \left( 1 - \frac{\bar{k}_y^i \xi_x}{\mu_x \varepsilon_y} - \frac{\bar{k}_z^i \xi_x}{\mu_y \varepsilon_x} \right)^{-1} \left[ P_{xa}^{n+1}_{i,j,k} + \frac{\bar{k}_y^i}{2\varepsilon_y} \left( Q_{ya}^{n+1}_{i,j+1,k} + Q_{ya}^{n+1}_{i,j,k} \right) - \\
\frac{\bar{k}_z^i}{2\varepsilon_x} \left( Q_{za}^{n+1}_{i,j+1,k} + Q_{za}^{n+1}_{i,j,k} \right) \right] \\
(5.22)
$$

$$
P_{y}^{n+1}_{i,j,k} = P_{ya}^{n+1}_{i,j,k} - \frac{\bar{k}_z^i}{2\varepsilon_y} \left( Q_{za}^{n+1}_{i,j+1,k} + Q_{za}^{n+1}_{i,j,k} \right) \\
(5.23)
$$

where $(i,j,k)$ represents the position of the spatial point, $n$ is the time step,
$\xi_x = \sigma_x \Delta t / \varepsilon_y \varepsilon_x$, $T_y = c \Delta t / \Delta y$, and $T_z = c \Delta t / \Delta z$. The modified temporal averaging of
the loss term is evident by the presence of $\beta$. The optimal choice for $\beta$ which results in
a maximum time steps that is the same as for the lossless case and satisfies the stability condition is given by [9]

\[ \beta_{opt} = \frac{0.5}{k_x^{1/2}} = \frac{0.5}{\sin^2 \theta \cos^2 \phi}. \]  

(5.24)

The singularities of the above equation are at the grazing angles where the system is unstable.

These equations present a full prescription of the split-field technique in three-dimensional in the main portion of the lattice. The stability relation of the system is described in the following section.

**B.2 Numerical Stability Analysis**

The stability relation of the split-field technique is derived by implementing the von Neumann analysis. This results in a 12×12 stability matrix. Setting determinant of this matrix to zero results in a twelfth-order polynomial that must be solved numerically to determine the stability limit [9].

However, the stability limit for the plane wave propagating in a general direction is bounded by two special cases \( k_y^i = k_z^i \) and \( k_y^i = 0 \) or \( k_z^i = 0 \), which they can be determined analytically. For the cubic Yee unit cells, along the direction \( k_y^i = k_z^i \) the stability limit is obtained as

\[ T_y \leq \frac{k_y^{i2}}{\sqrt{2 + k_x^{i2}}}. \]  

(5.25)

and for the case \( k_z^i = 0 \)
\[ T_y \leq \frac{k_x^{i2}}{k_y F_2 \sqrt{1 - F_2^2} + \left( k_x^{i2} + \frac{k_y^{i4}}{F_1} \right) \left( 2 + F_2^2 k_x^{i2} \right)} \]  

(5.26)

where

\[ F_1 = 8k_y^{i2} + k_x^{i6} + 2k_x^{i2}k_y^{i2} \]  

(5.27a)

\[ F_2^2 = \frac{(F_1 - k_x^{i2}k_y^{i2}) + \sqrt{(F_1 - k_x^{i2}k_y^{i2})^2 - 8F_1k_y^{i2}}}{2F_1}. \]  

(5.27b)

Now the stability factor for a general angle of incidence can be simply approximated by fitting a quadratic function of \( \phi_{yz} \) through the minimum and maximum points given by (5.25) and (5.26) [9]; where the value of angle \( \phi_{yz} \) is evaluated by

\[ \phi_{yz} = \cos^{-1}\left( \frac{k_y}{\sqrt{1 - k_x^{i2}}} \right). \]  

(5.28)

Although, compared to the usual FDTD technique the split-field method has less relaxed stability limit; however, as will be shown it is a very efficient and accurate method for broadband analysis of complex periodic structures.

5.1.4 Perfectly Matched Layer

Due to the finite computational resources available and the open boundary nature of the structures of interest, the computational domain in the FDTD technique must be artificially truncated. In this work, the truncation is performed using the anisotropic PML.
walls, which may absorb the EM waves without any reflection independent of the frequency, angle of incidence, and polarization of the waves.

A. Berenger’s PML Medium

PML Absorbing Boundary Conditions (ABC) introduced by Berenger [43] is an effective approach to truncate the FDTD computational domain. The idea is based on the creation of a nonphysical absorbing medium adjacent to the outer FDTD mesh boundary that has a wave impedance independent of the frequency, angle of incidence, and polarization of outgoing scattered waves.

In the PML media, each component of the electromagnetic fields is split into two parts [43] as, $E_{xy}, E_{xz}, \cdots, H_{zx}, H_{zy}$, satisfying the following equations

$$\varepsilon \frac{\partial E_{xy}}{\partial t} + \sigma_y E_{xy} = \frac{\partial (H_{zx} + H_{zy})}{\partial y}$$

(5.29a)

$$\varepsilon \frac{\partial E_{xz}}{\partial t} + \sigma_z E_{xz} = -\frac{\partial (H_{yx} + H_{zy})}{\partial z}$$

(5.29b)

$$\mu \frac{\partial H_{zx}}{\partial t} + \sigma_x^* H_{zx} = -\frac{\partial (E_{yx} + E_{zy})}{\partial x}$$

(5.29k)

$$\mu \frac{\partial H_{zy}}{\partial t} + \sigma_y^* H_{zy} = \frac{\partial (E_{xy} + E_{zy})}{\partial y}$$

(5.29l)

where the parameters ($\sigma_x, \sigma_y, \sigma_z, \sigma_x^*, \sigma_y^*, \sigma_z^*$) represent the electric and magnetic conductivities. The expression of each component of a propagating plane wave in the
PML media whose electric and magnetic fields form the angles $\phi_{ex}, \phi_{ey}, \phi_{ez}$, and $\phi_{hx}, \phi_{hy}, \phi_{hz}$ with the coordinate axis is represented in the following form

$$
\psi = \psi_0 e^{-j\omega(t-\alpha x-\beta y-\gamma z)}
$$

(5.30)

where $\alpha, \beta, \gamma$ are obtained as [43]

$$
\alpha = \sqrt{\varepsilon \mu G} \frac{w_{e} \cos \phi_{ey} \cos \phi_{hz} - w_{z} \cos \phi_{ez} \cos \phi_{hy}}{w_{x} \cos^{2} \phi_{hx} + w_{y} \cos^{2} \phi_{hy} + w_{z} \cos^{2} \phi_{hz}}
$$

(5.31a)

$$
\beta = \sqrt{\varepsilon \mu G} \frac{w_{x} \cos \phi_{ex} \cos \phi_{hx} - w_{z} \cos \phi_{ez} \cos \phi_{hx}}{w_{x} \cos^{2} \phi_{hx} + w_{y} \cos^{2} \phi_{hy} + w_{z} \cos^{2} \phi_{hz}}
$$

(5.31b)

$$
\gamma = \sqrt{\varepsilon \mu G} \frac{w_{y} \cos \phi_{ex} \cos \phi_{hy} - w_{x} \cos \phi_{ex} \cos \phi_{hx}}{w_{x} \cos^{2} \phi_{hx} + w_{y} \cos^{2} \phi_{hy} + w_{z} \cos^{2} \phi_{hz}}
$$

(5.31c)

and

$$
S_u = 1 + j \frac{\sigma_u}{\omega \varepsilon}, \quad S_u^* = 1 + j \frac{\sigma_u^*}{\omega \mu} \quad (u = x, y, z)
$$

(5.32)

$$
G = \sqrt{\frac{w_{x} w_{z} \cos^{2} \phi_{ex} + w_{x} w_{z} \cos^{2} \phi_{ey} + w_{x} w_{y} \cos^{2} \phi_{ez}}{w_{x} \cos^{2} \phi_{hx} + w_{y} \cos^{2} \phi_{hy} + w_{z} \cos^{2} \phi_{hz}}}
$$

(5.33)

$$
\frac{w_u}{S_u} = \frac{1 + j \sigma_u / \omega \varepsilon}{1 + j \sigma_u^* / \omega \mu} \quad (u = x, y, z).
$$

(5.34)

If the couples of conductivities $(\sigma_x, \sigma_x^*), (\sigma_y, \sigma_y^*), (\sigma_z, \sigma_z^*)$, satisfy the matching impedance condition $(\sigma / \varepsilon = \sigma^* / \mu)$, then each component of the propagating plane wave is simplified as follows

$$
\psi = \psi_0 e^{-j\omega(t-\alpha x-\beta y-\gamma z)} e^{-(\sigma_x \cos \phi_{hx} / \omega \varepsilon) x} e^{-(\sigma_y \cos \phi_{hy} / \omega \varepsilon) y} e^{-(\sigma_z \cos \phi_{hz} / \omega \varepsilon) z}
$$

(5.35)
where \( \vartheta_x, \vartheta_y, \) and \( \vartheta_z \) represent the angles that the perpendicular to the \((E, H)\) plane forms with the coordinate axis. The results show that in the PML media the phase propagates normal to the \((E, H)\) plane, and the magnitude decays exponentially in the \(x, y,\) and \(z\) directions.

The reflection coefficients for an interface normal to the \(x\)-axis between two anisotropic media with conductivities \((\sigma_{x1}, \sigma_{y1}, \sigma_{y1}, \sigma_{z1}, \sigma_{z1}^*)\) and \((\sigma_{x2}, \sigma_{x2}^*, \sigma_{y2}, \sigma_{y2}^*, \sigma_{z2}, \sigma_{z2}^*)\) for the TE (electric field lies in a plane parallel to the interface) and TM (magnetic field lies in a plane parallel to the interface) can be found to be, respectively,

\[
\Gamma_{TE} = \frac{Z_2 \cos \theta_1 - Z_1 \cos \theta_2}{Z_2 \cos \theta_1 + Z_1 \cos \theta_2} \quad (5.36a)
\]

\[
\Gamma_{TM} = \frac{Z_2 \cos \theta_2 - Z_1 \cos \theta_1}{Z_2 \cos \theta_2 + Z_1 \cos \theta_1} \quad (5.36b)
\]

where the wave impedance \(Z_n\) \((n=1 \text{ or } 2)\) in each region is

\[
Z_n = \sqrt{\mu / \varepsilon (1 / G_n)} \quad (5.37)
\]

Additionally, the Snell’s law on the interface can be written as

\[
\beta_1 = \beta_2, \quad \gamma_1 = \gamma_2 \quad (5.38)
\]

In the particular case, when two media having the same transverse conductivities, the same permittivities and permeabilities, and moreover they are also matched \((\sigma / \varepsilon = \sigma^*/\mu)\), from Eqs. (5.37) and (5.38) is obtained that \(Z_1 = Z_2\) and \(\theta_1 = \theta_2\), and then the reflection coefficients for both TE and TM cases become equal to zero,
independent of the frequency and incident angle. Obviously these properties are also valid for the interfaces normal to the $y$ and $z$ directions.

Therefore, with the PML material described above, a very lossy, non-reflective material can be implemented in the FDTD simulation. By surrounding the structure of interest with such non-reflective absorbing material, the computational domain can be simply truncated in a Perfect Electric Conductor (PEC). Fig. 5.4 depicts the upper-right part of a computational domain surrounded by the PML layers with the proper conductivities. Note that, in the corners of the PML (overlapping regions), suitable conductivities for the connected materials are used to effectively absorb the EM waves.

The outgoing waves from the inner region penetrate without reflection into the PML ABC layers. In the PML media, the magnitude of the wave decays exponentially with the distance $\rho$ as (Eq. (5.35))

$$\psi(\rho) = \psi(0)e^{-(\sigma \cos \theta/\epsilon)\rho}$$  \hspace{1cm} (5.39) $$

where $\theta$ is the incident angle with respect to the normal on the interface. This wave is reflected back by the PEC, which ends the domain and it can return to the inner region with the reflection factor

$$R(\theta) = e^{-2(\sigma \cos \theta/\epsilon)\delta}$$ \hspace{1cm} (5.40) $$

where $\delta$ is the PML thickness. Note that if the outgoing wave is close to the PML interface ($\theta = \pi/2$), the factor $R$ is close to the unity, whatever $\sigma$ may be. This has not been a problem in the FDTD simulation, since such a wave is near normal on the perpendicular PML boundaries and is absorbed. As a result, no significant reflected energy will exist after the round trip through the PML materials. To avoid a sharp
variation of the conductivity, as Berenger proposed, a gradual loss PML medium has to be used.

Although PML attenuates the propagating waves, it doesn’t attenuate the evanescent waves of the near fields of the structure of interest. Because of this a buffer region must be introduced between the structure and the PML boundary to ensure the terminating PEC doesn’t perturb the non-radiating near fields.

Fig. 5.4: Upper-right part of the computational domain surrounded by the PML layers. The PML is truncated by the PEC.
B. Anisotropic PML Medium

The PML introduced by Berenger is a hypothetical medium based on a mathematical model. Due to the coordinate dependence of the loss term, it represents an anisotropic material. In fact, it is obtained that a Uniaxial Perfectly Matched Layer (UPML) \[9\] has the similar propagation characteristics as the Berenger’s PML and it can absorb the EM waves without any reflection.

In general, all regions of the FDTD space can be simply modeled using the Eqs. (5.3) and (5.4). In the PML region the tenors $\bar{\varepsilon} = \varepsilon$, $\bar{\mu} = \mu$, and $\bar{s}$ is \[9\]

$$
\bar{s} = \begin{bmatrix}
    s_x & s_z & 0 & 0 \\
    s_x & s_z & 0 & 0 \\
    0 & s_y & s_x & 0 \\
    0 & 0 & s_y & s_z \\
\end{bmatrix}
$$

(5.41)

where

$$
s_u = 1 + \frac{\chi_u c}{j \omega}, \quad \chi_u = \frac{\sigma_u \eta_0}{\varepsilon_r}, \quad (u = x, y, z).
$$

(5.42)

Note that the tensor $\bar{s}$ includes the effect of both interior and the corner regions (the region where the PML media overlap) of the PML. In the nonoverlapping regions it is reduced to a uniaxial matrix.
C. Imposing PBC in the UPML Medium

Periodic structures may require extensive simulation time due to the multiple reflections, which occur in setting up the modal distribution on the periodic body. Therefore, the absorbing boundary wall may generate significant error due to the multiple reflections. The PML ABC has the great capability in truncating the periodic structures.

To formulate the problem, the structure with periodicity in the \( y-z \) directions (Fig. 5.3) is terminated to the PBC walls in the periodic sides and UPML in the \( x \) direction. In the UMPL medium, the uniaxial tensor \( \bar{s} \) is presented as

\[
\bar{s} = \begin{bmatrix} 1/s_x & 0 & 0 \\ 0 & s_x & 0 \\ 0 & 0 & s_x \end{bmatrix}.
\]  

(5.43)

Now the Maxwell’s equations for the transformed field variables \( \mathbf{P} \) and \( \mathbf{Q} \) are given by

\[
\frac{j \omega \varepsilon_r}{c} \hat{P}_x = \frac{\partial Q_z}{\partial y} - \frac{\partial Q_y}{\partial z} - \frac{j \omega k_x}{c} Q_z + \frac{j \omega k_y}{c} Q_y
\]  

(5.44a)

\[
\frac{j \omega \varepsilon_r s_x}{c} \hat{P}_y = -\frac{\partial Q_z}{\partial x} + s_x \frac{\partial Q_y}{\partial z} - \frac{j \omega k_x}{c} Q_z
\]  

(5.44b)

\[
\frac{j \omega \varepsilon_r s_x}{c} \hat{P}_z = \frac{\partial Q_y}{\partial x} - s_x \frac{\partial Q_z}{\partial y} + \frac{j \omega k_y}{c} Q_z
\]  

(5.44c)

\[
\frac{j \omega \mu_r}{c} \hat{Q}_x = -\frac{\partial P_z}{\partial y} + \frac{\partial P_y}{\partial z} + \frac{j \omega k_y}{c} P_z - \frac{j \omega k_x}{c} P_y
\]  

(5.45a)

\[
\frac{j \omega \mu_r s_x}{c} \hat{Q}_y = \frac{\partial P_z}{\partial x} - s_x \frac{\partial P_z}{\partial y} + \frac{j \omega k_y}{c} P_z
\]  

(5.45b)

\[
\frac{j \omega \mu_r s_x}{c} \hat{Q}_z = -\frac{\partial P_y}{\partial x} + s_x \frac{\partial P_y}{\partial y} - \frac{j \omega k_y}{c} P_y
\]  

(5.45c)
where $\hat{P}_x = P_x/s_x$ and $\hat{Q}_x = Q_x/s_x$. By defining new variables as was done for the non-PML case the extra time derivatives in the right-hand side are eliminated. The EM fields are now split in the following form

\[
\hat{P}_x = P_{xa} + \frac{k^i_y}{\varepsilon_r} Q_y - \frac{k^i_z}{\varepsilon_r} Q_z \quad (5.46a)
\]

\[
P_y = P_{ya} + \frac{c}{j \omega \varepsilon_r} \frac{\partial \hat{Q}_x}{\partial z} - \frac{k^i_z}{\varepsilon_r} \hat{Q}_y, \quad P_{yb} = \frac{c}{j \omega \varepsilon_r} \frac{\partial \hat{Q}_x}{\partial z} \quad (5.46b)
\]

\[
P_z = P_{za} - \frac{c}{j \omega \varepsilon_r} \frac{\partial \hat{Q}_x}{\partial y} + \frac{k^i_y}{\varepsilon_r} \hat{Q}_y, \quad P_{zb} = -\frac{c}{j \omega \varepsilon_r} \frac{\partial \hat{Q}_x}{\partial y} \quad (5.46c)
\]

\[
\hat{Q}_x = Q_{xa} - \frac{k^i_z}{\mu_r} P_y + \frac{k^i_y}{\mu_r} P_z \quad (5.47a)
\]

\[
Q_y = Q_{ya} - \frac{c}{j \omega \mu_r} \frac{\partial \hat{P}_x}{\partial z} + \frac{k^i_z}{\mu_r} \hat{P}_y, \quad Q_{yb} = -\frac{c}{j \omega \mu_r} \frac{\partial \hat{P}_x}{\partial z} \quad (5.47b)
\]

\[
Q_z = Q_{za} + \frac{c}{j \omega \mu_r} \frac{\partial \hat{P}_x}{\partial y} - \frac{k^i_y}{\mu_r} \hat{P}_y, \quad Q_{zb} = \frac{c}{j \omega \mu_r} \frac{\partial \hat{P}_x}{\partial y} \quad (5.47c)
\]

Substituting these into the (5.44) and (5.45) yields the time domain equations for the split fields

\[
\frac{\varepsilon_r}{c} \frac{\partial P_{xa}}{\partial t} = \frac{\partial Q_y}{\partial y} - \frac{\partial Q_z}{\partial z} \quad (5.48a)
\]

\[
\varepsilon_r \frac{\partial P_{ya}}{\partial t} + \varepsilon_r \chi_x P_{ya} = -\frac{\partial Q_x}{\partial x}, \quad \frac{\varepsilon_r}{c} \frac{\partial P_{yb}}{\partial t} = \frac{\partial \hat{Q}_x}{\partial z} \quad (5.48b)
\]

\[
\varepsilon_r \frac{\partial P_{za}}{\partial t} + \varepsilon_r \chi_x P_{za} = \frac{\partial Q_x}{\partial x}, \quad \frac{\varepsilon_r}{c} \frac{\partial P_{zb}}{\partial t} = -\frac{\partial \hat{Q}_x}{\partial y} \quad (5.48c)
\]
Once the “a” and “b” portions of the fields (components of $P$ and $Q$ with the indices “a” and “b”) are determined, then the normal components to the PML are obtained using the Eqs. (5.46) and (5.47)

\[ \begin{align*}
\left( 1 - \frac{k_x^2}{\mu, \varepsilon_r} - \frac{k_z^2}{\mu, \varepsilon_r} \right) \hat{P}_x &= P_{xa} + \frac{k_x^i}{\mu} (Q_{ya} + Q_{yb}) - \frac{k_z^i}{\varepsilon_r} (Q_{za} + Q_{zb}) \\
\left( 1 - \frac{k_x^2}{\mu, \varepsilon_r} - \frac{k_z^2}{\mu, \varepsilon_r} \right) \hat{Q}_x &= Q_{xa} - \frac{k_x^i}{\mu} (P_{ya} + P_{yb}) + \frac{k_z^i}{\varepsilon_r} (P_{za} + P_{zb}).
\end{align*} \] (5.50a)

The tangential fields on the PML are obtained as

\[ \begin{align*}
P_y &= P_{ya} + P_{yb} - \frac{k_x^i}{\varepsilon_r} \hat{Q}_x \\
P_z &= P_{za} + P_{zb} + \frac{k_x^i}{\varepsilon_r} \hat{Q}_x \\
Q_y &= Q_{ya} + Q_{yb} + \frac{k_x^i}{\mu} \hat{P}_x \\
Q_z &= Q_{za} + Q_{zb} - \frac{k_x^i}{\mu} \hat{P}_x.
\end{align*} \] (5.51a)
Note that for the free-space/PML interface the values of $\varepsilon$, and $\mu$, are equal to one. Compared to the non-PML region, additional splitting for the tangential fields in the UPML is performed to make the system stable.

In developing this technique both local stability and simulation involving many time steps are used to verify the stability of the system.

### 5.1.5 Prony’s Extrapolation Scheme

As mentioned earlier periodic structures may require extensive simulation time due to the multiple reflections, which occur in setting up the modal distribution on the periodic body. Thus, the simulation involves many time steps, which is time consuming and it increases the numerical errors over the long period of time. There is a tremendous advantage in both efficiency and accuracy of the FDTD analysis if the computational time can be effectively truncated.

The Prony’s method [44] is an extrapolation scheme, which is applied to truncate the time domain analysis and record the FDTD data over a much shorter time. It has a potential advantage in analyzing the periodic structures in order to achieve the broadband results accurately.

In the first step of extrapolation scheme, the structure is analyzed using the FDTD technique and the results are recorded in a relatively short period of time. Next, the Prony’s method is used to fit a set of complex exponential functions to the FDTD data and obtain the oscillation frequencies and damping factors, which can properly represent
the behavior of the resonance structures. The sampled FDTD time records $d_n$ ($d_n$ could be any of the field components) are approximated as

$$d_n = \sum_{m=1}^{M} a_m z_m^n, \quad n = 1,2,\cdots, N$$  \quad (5.53)

where $N$ and $M$ are the number of data and fitting functions ($N \geq M$), respectively, $a_m$ is the complex coefficient, and $z_m$ is the complex exponential function including both oscillation frequency and damping factor. It is readily verified that $d_n$ satisfies the difference equation [44]

$$d_n = -\sum_{m=1}^{M} b_m d_{n-m}, \quad n = M + 1, M + 2,\cdots, N$$  \quad (5.54)

where $b_m, m = 1,2,\cdots, M$ are the coefficients of the polynomial

$$g(z) = z^M + b_1 z^{M-1} + \cdots + b_M$$  \quad (5.55)

which has complex roots $z_1, z_2,\cdots, z_M$ for representing $d_n$ as given in (5.53).

A least-square method is applied to solve the over determined Eq. (5.54) and evaluate the coefficients $b_m$. Next, these coefficients are used in (5.55) to determine the complex roots $z_m$. The final step is to substitute the roots $z_m$ in (5.53) and solve for the resulting least square equation to obtain the unknown coefficients $a_m$.

The results for the future time steps can now be evaluated very efficiently using the Eq. (5.53) while bypassing the time consuming FDTD method; and it is not unusual to realize a reduction in the computational time from hours to seconds. For example, it is demonstrated in Appendix B that to accurately analyze a double concentric square loop
FSS one has to apply the FDTD technique for about 28 cycles (155 min.); however, utilizing the Prony’s scheme, the FDTD is used only in the first 10 cycles (55 min.) and the results for the next 18 cycles are efficiently determined using the closed form Eq. (5.53) in about 0.04 min. (this takes about 100 min. based on the FDTD technique). The time saving and accuracy of the FDTD/Prony method generates a powerful engine in broadband analysis of complex periodic structures. Notice that, to successfully apply the Prony’s method one needs to properly truncate the time domain results as discussed in Appendix B.

5.2 Characterization of Challenging EBG Structures

In this chapter, the development of a powerful computational engine utilizing FDTD technique with periodic boundary condition/perfectly matched layer is presented to explore and obtain the innovative propagation characteristics of the complex and challenging periodic EBG structures.

The Floquet transformed Maxwell’s equations are discretized using the split-field approach. Taking advantage of the broadband analysis of the FDTD technique provide great efficiency and accuracy when the structure is characterized to demonstrate its frequency response. The Prony’s extrapolation scheme is also incorporated to increase the efficiency of the technique. Utilizing the developed engine, one can place the PBC walls on the PEC/dielectric sides of the structure, which may be very useful in the analysis of some of the complex periodic structures.
The objective in the following chapters is to investigate the electromagnetic performance of the different classes of complex EBG structures namely, (a) FSS structures, (b) PBG materials, (c) smart surfaces for communication antenna applications, (d) surfaces with perfectly magnetic conducting properties (PMC), (e) surfaces with reduced edge diffraction effects, (f) periodic layered dielectric materials, and (g) composite materials with negative permittivity and negative permeability. The results are incorporated into some potential applications.

In this work, to successfully apply the FDTD to the analysis of a complex periodic structure, the computational domain is filled using the Yee unit cells with the cell size around $0.01\lambda$ ($\lambda$ is the wavelength at the center frequency of the incident wave), and the domain is truncated to the PBC/PML walls. The PBC is positioned in the $y$-$z$ directions, and a 10-cell-thick PML with the reflection coefficient $R(0) = 10^{-7}$ is located in the $x$ direction. A separation distance about $1\lambda$ between the periodic structure and PML is used to allow a sufficient space for the evanescent modes to die before encountering the PML.
Chapter 6

Frequency Selective Surfaces

Frequency selective surfaces composed of complex periodic scatterers of dielectrics and conductors of arbitrary shapes have many applications in different areas of electromagnetics and satellite communications [45]-[54]. These structures can completely reflect the incident plane waves in some ranges of frequency, while they can be totally transparent to the EM waves in other ranges. This depends on the shape of the basis resonance element itself and the lattice on which the elements are arranged. Since different structural configurations radically change the characteristics of the structure, the geometry may be cleverly used to tailor the frequency response for interesting applications such as filter designs and band-gap structures. Due to the vector nature of the electric and magnetic fields, this response depends on the angle as well as the polarization of the incident waves.

The focus of this chapter is to characterize and understand the performance of different types of complex FSS. The FDTD/Prony technique is applied to present the normal and oblique incident reflected or transmitted plane wave frequency response. The results are compared very well with the available analytical, numerical, and measured
data; which it demonstrates the capability of the developed technique in the analysis of complex periodic structures.

6.1 Multi-Layered Planar Dielectric Structures

As the first example, in order to show the capability and accuracy of the developed technique, a multi-layered planar dielectric structure is analyzed. Fig. 6.1 depicts the geometry and parameters of a 5-layer infinite dielectric structure. To model the infinite structure, the PBC walls are positioned on the dielectric sides of the unit cell (Fig. 6.1b). The FDTD domain is terminated to the PML in the $x$ direction.

The broadband FDTD analysis of the reflection coefficients for the normal, 60° oblique/TE ($E_x = 0$), and 60° oblique/TM ($H_x = 0$) incident plane waves are presented in Figs. 6.2-6.4. The results are compared with the analytic solution derived from the vector wave solution of Maxwell’s equations, as discussed in Chapter 2. The excellent agreements between the modal solution and FDTD technique validate the accuracy and capability of the FDTD computational engine.

It is observed that, the periodic like dielectric contrast between the slabs produces an almost band-gap region for the TE waves. This phenomenon is detailed more in the next chapter.
Fig. 6.1: 5-Layer dielectric structure: (a) Infinite size dielectric in the y-z directions, (b) Unit cell of the structure truncated to the PBC/PML walls.

<table>
<thead>
<tr>
<th>Layer</th>
<th>$\varepsilon_r$</th>
<th>t (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.17</td>
<td>3.0</td>
</tr>
<tr>
<td>2</td>
<td>2.80</td>
<td>1.80</td>
</tr>
<tr>
<td>3</td>
<td>1.20</td>
<td>6.00</td>
</tr>
<tr>
<td>4</td>
<td>2.80</td>
<td>1.80</td>
</tr>
<tr>
<td>5</td>
<td>6.17</td>
<td>3.00</td>
</tr>
</tbody>
</table>

Fig. 6.2: Normal incidence reflection coefficient of the multi-layered dielectric structure. FDTD compared to the analytic vector wave solution.
Fig. 6.3: 60° oblique incidence reflection coefficient of the multi-layered dielectric structure. FDTD compared to the analytic vector wave solution (TE case).

Fig. 6.4: 60° oblique incidence reflection coefficient of the multi-layered dielectric structure. FDTD compared to the analytic vector wave solution (TM case).
6.2 Double Concentric Square Loop FSS

A periodic structure of double concentric square loop FSS is presented in Fig. 6.5. Conductors with thickness $t$ and square cross sections form the loops. The structure has the potential to reflect the EM waves in some ranges of frequency while it is transparent in other ranges. The percentage of the reflected power for the normal and 60° oblique/TM plane waves is computed utilizing the FDTD technique and presented in Figs. 6.6 and 6.7. The results are obtained based on the sin/cos and split-field methods and compared very well with the reference data reported in [9]. It is demonstrated that the designed FSS produces a frequency response with both stop-band (completely reflect) and pass-band (completely transmit) regions in the frequency range of the incident wave.

As illustrated in Fig. 6.6, for the normal incidence, the split-field and sin/cos approaches agree best, but there is a very small shift compared to the MoM data. In the oblique case displayed in Fig. 6.7, the split-field results are slightly shifted compared to the sin/cos and MoM techniques, but in general, a very good agreement is retained. Also as observed the developed FDTD is able to effectively determine the locations of the resonance frequencies of the periodic structure for both normal and oblique cases. The results successfully prove the capability of the developed FDTD technique. Note that the split-field method is a broadband analysis, which compared to the single frequency approaches is much more efficient in obtaining the frequency response of the complex structures.

In this case, the comparison between the split-field and split-field/Prony methods is also explored, and as shown an excellent agreement is obtained. In the FDTD/Prony
technique, the time domain analysis is truncated and the results are efficiently computed in a shorter interval of time (Appendix B).

Fig. 6.5: Double concentric square loop FSS: (a) Periodic structure, (b) Unit cell of the structure.

Fig. 6.6: Normal incidence reflected power of the double concentric square loop FSS. FDTD/Prony compared to the split-field, sin/cos, and MoM techniques [9].
6.3 Dipole FSS Manifesting High $Q$ Resonance

A periodic array of dipole FSS as shown in Fig. 6.8 is analyzed in this section. The structure is in principle opaque at the resonance frequencies, while is transparent somewhat below and above the resonance regions. The reflection coefficient for the normal and oblique incidence plane waves are determined and presented in Fig. 6.9. The electric field is polarized along the length of the dipoles (TM case). In the computation, the Prony’s method is also integrated to efficiently obtain the results. The FDTD/Prony results closely resemble the MoM data presented in [55].

For the oblique incident, the reflection coefficient is calculated for two different cases: $\theta^i = 89^\circ, \phi^i = 180^\circ$ and $\theta^i = 90^\circ, \phi^i = 179^\circ$, which they have 1° difference
compared to the normal case ($\theta^i = 90^\circ, \phi^i = 180^\circ$). In the first case, the plane wave is tilted along the narrow width of the dipoles and as expected almost similar performance with the normal case is achieved (Fig. 6.9). However, for the $\theta^i = 90^\circ, \phi^i = 179^\circ$, the tilt angle is along the length of the dipoles, which may vary the current distribution on the FSS resulting in the change of frequency response. As demonstrated in Figs. 6.9 and 6.10, in this case compared to the normal case, a high $Q$ (quality factor) resonance behavior around the $1 - \lambda$ dipole length is obtained.

The resonant current (for the dipoles $\approx 1\lambda$) along the dipole length has an odd distribution [55]. For the normal incidence, a complete symmetry exists and no resonance behavior is observed. In the oblique case, since there is not an exact cancellation of the dipole currents, the structure is able to resonate. To obtain a complete reflection the amplitude of the current must be large enough; however, in practice, for a real metal with finite conductivity the dipole FSS cannot support an arbitrary large current distribution and the high $Q$ resonance phenomena is not observed [55].

The analysis of the $1 - \lambda$ dipole FSS, manifesting high $Q$ resonance, based on the frequency domain methods is a very difficult task. To capture the high $Q$ resonance, the simulation has to be performed in a tremendous number of frequency points, which is very time consuming. However, the FDTD technique is a broadband analysis, which can effectively determine the high $Q$ resonance frequencies. The only drawback is that now a simulation involving many time steps are required to obtain an accurate frequency response. For instance, the computer run time for the dipole FSS with high $Q$ resonance behavior on an HPC-180 workstation is estimated to around 25 days. Utilizing
FDTD/Prony, the reflection coefficient is accurately determined in about 10 hr., which is extremely efficient.

![Fig. 6.8: Dipole FSS manifesting high $Q$ resonance: (a) Periodic structure, (b) Unit cell of the structure.](image)

Note that the PBC walls are positioned on the PEC edges.

![Fig. 6.9: Normal and oblique incidence reflection coefficient of the dipole FSS. Note the anomalous behavior around the $1-\lambda$ dipole length for the $1^\circ$ tilt angle along the length.](image)
Fig. 6.10: Resonance behavior of the $1-\lambda$ dipole FSS for the $1^\circ$ oblique incidence wave (tilt along the length of dipoles). The performance is presented in an expanded range around the resonance frequency.

6.4 Patch FSS Manifesting High $Q$ Resonance

In this section the performance of another FSS with high $Q$ resonance behavior is obtained. The reflection coefficient for a periodic array of square patch FSS as displayed in Fig. 6.11, is presented in Fig. 6.12. The electric field is polarized along the $z$ direction (TE case). Results are obtained based on the FDTD/Prony technique, and as observed for the oblique incidence at $\theta^i = 90^\circ, \phi^i = 179^\circ$, a $1/2-\lambda$ high $Q$ resonant performance is determined. The slight asymmetry in the odd current distribution on the patch FSS provides the resonance performance [55]. Fig. 6.13 shows the behavior of the FSS around
the resonance frequency. It is obtained that a pass-band occurs around the $1/2 - \lambda$ patch size. The results agree well with the data in [55] utilizing MoM.

Fig. 6.11: Square patch FSS manifesting high $Q$ resonance: (a) Periodic structure, (b) Unit cell of the structure.

Fig. 6.12: Normal and oblique incidence reflection coefficient of the patch FSS. Note the anomalous behavior around the $1/2 - \lambda$ patch size for the $1^\circ$ oblique incidence.
Fig. 6.13: Resonance behavior of the $1/2 - \lambda$ patch FSS for the $1\degree$ oblique incidence wave. The performance is presented in an expanded range around the resonance frequency.

6.5 Crossed Dipole FSS

The efforts to control pass-/stop-band characteristics of the periodic structures have led to the development of the crossed dipole FSS. The shape of the structure allows one to effectively control the rate of transition from pass-band to the stop-band [56]. Fig. 6.14 shows a crossed dipole FSS printed on the dielectric material. The FDTD/Prony technique is applied to obtain the transmission coefficient for the TM case oblique incident at $30\degree$, as presented in Fig. 6.15. It is observed that the structure has two narrow stop-bands at the frequencies 30.8 GHz ($-38 dB$ transmission) and 34.0 GHz ($-8.5 dB$ transmission). The results have a very good agreement with the measurement data [56].

Notice that, the analysis of the periodic structures on the dielectric materials using
the frequency domain methods is a very difficult task; while our capable FDTD/Prony technique has the potential to simply and accurately characterize the metallic/dielectric periodic structures.

Fig. 6.14: Crossed dipole FSS: (a) Periodic structure, (b) Unit cell of the structure. Note that the PBC walls are positioned on the dielectric sides.

Fig. 6.15: 30° oblique incidence transmission coefficient of the crossed dipole FSS. Note the good agreement with the measurement data [56].
6.6 Dichroic Plate FSS

In this section the FDTD analysis of the X-/Ka-band dichroic plate FSS \[57\] including square holes in the thick PEC, as displayed in Fig. 6.16 is obtained. The reflection coefficients for the normal ($E_z$) and TE/TM-30° ($\theta' = 90°, \phi' = 150°$) oblique incident plane waves are determined in Fig. 6.17. The performance of the structure in the expanded frequency range $31–36 \, GHz$ is also shown in Fig. 6.18.

As observed, the dichroic plate FSS is transparent (about $-20 \, dB$ reflection coefficient) to the Ka-band downlink ($31–32.3 \, GHz$), while at the same time reflecting the X-band region ($8–12 \, GHz$). The holes on the dichroic plate are large enough to cover the lower frequency end of the Ka-band and they are tightly packed to prevent the grating lobes at the high frequency end. There is a good agreement with the data presented in \[57\].

![Dichroic Plate FSS](image)

Fig. 6.16: Dichroic plate FSS: (a) Periodic structure, (b) Unit cell of the structure. Note that the PBC walls are positioned on the PEC edges.
Fig. 6.17: Normal and 30° oblique (TE/TM) incidence reflection coefficient of the dichroic plate FSS.

Fig. 6.18: Performance of the dichroic plate FSS in the expanded frequency range 31–36 GHz for the normal and 30° oblique (TE/TM) incidence plane waves.
6.7 Inset Self-Similar Aperture FSS

The focus of this section is to characterize a dual-band inset self-similar aperture FSS printed on the dielectric substrate, as shown in Fig. 6.19, utilizing the FDTD/Prony method. The FSS is constructed from two periodic structures namely, small square patches and large size square apertures. The characteristics of the small patches (structure in Fig. 6.19 without metallic strips), large apertures (structure in Fig. 6.19 without patches), and the composite square patch/square aperture are demonstrated in Fig. 6.20.

As observed, the periodic patches resonant at the high frequencies (stop-band) while the periodic apertures are able to reject the d.c. frequencies. The composition of these two periodic elements produces a dual-band FSS, illustrated in Fig. 6.20. The FDTD results are compared very well with the measurement data determined in the UCLA antenna Lab. There is about 60 GHz frequency isolation between the stop-bands of the designed FSS.

The behavior of the structure for the 30° (θ' = 90°, φ' = 150°) TE/TM oblique incidence plane waves is also explored in Fig. 6.21. As obtained, the performance of the FSS is sensitive to the polarization and angle of incidence. The TE and TM waves have different resonance frequencies.
Fig. 6.19: Inset self-similar aperture FSS: (a) Periodic structure, (b) Unit cell of the structure. Note that the PBC walls are positioned on the PEC/dielectric sides.

Fig. 6.20: Normal incidence transmission coefficient of the square patch, square aperture, and inset self-similar aperture based on the FDTD compared to the measurement data.
Fractal FSS

Fractal shaped antennas are a new class of self-similar structures with unique characteristics that are applied into some novel applications. Self-similarity of the fractals can be described as the replication of the geometry of the structure in different scales within the same structure. This has a potential advantage in the design of multi-band frequency selective surfaces [53]. Additionally, the unique shape of the fractal antenna allows one to confine an arbitrary long curve antenna in a given small area. This property is very useful in reducing the spacing between the resonance elements in an FSS, and in reducing the area occupied by the small antennas [58], [59]. This may have the potential applications in the design of miniaturized antennas.
This section focuses on the characterization of a two-iteration Sierpinski dipole FSS utilizing the powerful FDTD/Prony engine. The structure is presented in Fig. 6.22. The transmission coefficient of the free standing fractal FSS for the normal incidence plane wave \( E_z \) is displayed in Fig. 6.23. As obtained, the structure has two resonance frequencies with the total reflection at 5.68 and 21.39 GHz.

The dielectric backed fractal FSS is also analyzed in this section and the results for both normal and 30° \( (\theta' = 90°, \phi' = 150°) \) oblique incidence (TE case) are determined in Fig. 6.24. The resonance frequencies for the normal case are located around 3.89 and 15 GHz which are within a 5% error compared to the measurement data [53]. As observed, the resonance frequencies of the dielectric backed fractal FSS are shifted down compared to the free standing fractal. The resonance frequencies of the FSS on the dielectric material can be simply estimated using the following equation

\[
f_d = f_0 / \sqrt{\varepsilon_{ave}}, \quad \varepsilon_{ave} = \frac{\varepsilon_r + 1}{2}
\]

(6.1)

where \( f_0 \) and \( f_d \) are the resonance frequencies of the free standing and dielectric backed FSS, respectively. The estimated values are around 4.23 and 15.94 GHz, which are close to the computed results.

Although, the FDTD with the rectangular grids are applied to approximate the triangular edges of the Sierpinski fractal, but still the good agreements with the presented data in [53] are obtained. Notice that the staircasing with cell size 0.01\( \lambda \) (\( \lambda \) is the center frequency) is used to approximate the triangular boundaries.
Fig. 6.22: Sierpinski fractal FSS: (a) Free standing fractal, (b) Dielectric backed fractal, (c), (d) Unit cells of the structures. Note that the PBC walls are positioned on the sides of the structures.

Fig. 6.23: Normal incidence transmission coefficient of the free standing Sierpinski fractal FSS.
6.9 Multi-Layered Tripod FSS

In this section, the performance of a multi-layered tripod FSS band-gap structure is studied. The geometry of a single layer tripod structure is shown in Fig. 6.25. The FDTD/Prony technique is applied to compute the normal ($E_z$) and $30^\circ$ TE/TM oblique incidence ($\theta^i = 90^\circ, \phi^i = 150^\circ$) reflected power, as presented in Fig. 6.26. The results are similar with the data obtained in [54] utilizing MoM. To compare the results with the measurement data [55], the transmission coefficient at the normal incidence for the tripod structure printed on the dielectric substrate is determined in Fig. 6.27. There is an excellent agreement with the measurement data validating the accuracy and capability of the developed technique.
The location of the resonance frequency of the tripod structure can be controlled using a 2-layer tripod, as presented in Fig. 6.28. The second layer is rotated $180^\circ$ with respect to the first layer, and is shifted along the $z$-axis in such a way that all three legs of each tripod overlap a leg of a tripod in the first layer. The overlap region forms a capacitor, which is used to tune the frequency for 100% reflection. The reflected power of the 2-layer tripod is determined in Fig. 6.29. As observed, compared to the 1-layer tripod the resonance frequency is shifted down.

To broaden the 100% reflection bandwidth of the tripod structure, a 4-layer tripod as depicted in Fig. 6.30 (composition of two sets of the structure shown in Fig. 6.28) is introduced. The geometry has two degrees of freedom, $d$ and $D$. The capacitance governed by the small distance $d$ controls the lower edge of the rejection band, where the large inter-capacitor spacing $D$ controls the upper edge of the band. The reflected power is presented in Fig. 6.31. As observed, by increasing the bandwidth of the 100% reflection, a band-gap structure utilizing the multiple coupled tripod arrays is designed. The results agree well with the data presented in [54].

Notice that for the angles near to the grazing, the TM waves are almost normal to the surface of tripods and the FSS cannot reject them. Utilizing the interconnecting vias between tripods may help to reject the normal components and design a complete band-gap structure as achieved in [54].

In this example, the inter-spacing $D$ is much greater than the distance $d$ between the tripods in each set, and one needs a powerful engine to analyze the structure. The accurate and efficient results presented in this section, based on the FDTD/Prony method,
show the capability of the developed technique in the characterization of complex structures.

Fig. 6.25: 1-Layer tripod structure: (a) Free standing tripod, (b) Dielectric backed tripod, (c), (d) Unit cells of the structures. Note that the PBC walls are positioned on the sides of the structures.
Fig. 6.26: Normal and $30^\circ$ oblique (TE/TM) incidence reflected power of the 1-layer tripod FSS.

Fig. 6.27: Normal incidence transmission coefficient of the 1-layer tripod FSS printed on the dielectric substrate.
Fig. 6.28: 2-Layer tripod structure: (a) Periodic structure, (b) Unit cell of the structure. Note that the PBC walls are positioned on the PEC edges.

Fig. 6.29: Normal and 30° oblique (TE/TM) incidence reflected power of the 2-layer tripod FSS.

$$W = 0.40 \text{ mm}$$
$$d = 20 \mu \text{m}$$
$$T_x = 1.40 \text{ mm}, T_z = 2.44 \text{ mm}$$
Fig. 6.30: 4-Layer tripod structure (composition of two sets of the structure shown in Fig. 6.28): (a) Periodic structure, (b) Unit cell of the structure. Note that the PBC walls are positioned on the PEC edges.

Fig. 6.31: Normal and $30^\circ$ oblique (TE/TM) incidence reflected power of the 4-layer tripod EBG structure.

$W = 0.40 \text{ mm}$
$d = 20 \mu \text{m}$
$D = 1 \text{ mm}$
$T_y = 1.40 \text{ mm}, T_z = 2.44 \text{ mm}$
6.10 9-Layer Dielectric/Inductive Square Loop FSS

A 9-layer dielectric/inductive square loop pass-band FSS, designed by Munk [11], is characterized in this section. The structure is the composition of 2-layer cascaded inductive square loop FSS sandwiched between the 7 layer dielectric materials, as depicted in Fig. 6.32. The broadband analysis of the complex structure is obtained using the FDTD/Prony technique.

The transmission coefficients of the 2-layer square loop, 7-layer dielectric material, and the composite square loop/dielectric material for the normal and 30° TE/TM oblique incidence ($\theta^i = 90°, \phi^i = 150°$) plane waves are presented in Figs. 6.33, 6.34, and 6.35, respectively.

The inductive square loop FSS has the property to pass the EM waves at the resonance frequency. To generate a frequency response with an almost flat-top or gap region, 2 layers of the FSS are cascaded, as obtained in Fig. 6.32a with the transmission coefficient in Fig. 6.33. Additionally as observed in Fig. 6.34, the properly layered dielectric materials have the potential to effectively produce the pass/stop band regions for the EM waves. Sandwiching the cascaded FSS layers between the dielectric slabs (Fig. 6.32c) presents a novel metallo-dielectric periodic structure with the improved electromagnetic gap region, as demonstrated in Fig. 6.35. The composite structure provides a frequency performance with an almost constant bandwidth and flat gap region for the plane waves incident on the structure. Increasing the number of FSS layers and using more suitable layered dielectric materials may improve the band-gap characteristics of the structure.
Fig. 6.32: 9-Layer dielectric/inductive square loop FSS: (a) 2-Layer inductive square loop, (b) 7-Layer dielectric material, (c) Composite square loop/dielectric material (9 layers), (d) Cross section of the layered structure, (e) Unit cell of the square loop FSS.

#1: \(\varepsilon_i = 4.50, t_i = 7.92 \text{ mm}\)
#2: \(\varepsilon_j = 2, t_j = 0.99 \text{ mm}\)
#3: Inductive Square Loop
#4: \(\varepsilon_k = 2, t_k = 1.49 \text{ mm}\)
#5: \(\varepsilon_l = 14, t_l = 3.96 \text{ mm}\)
#6: \(\varepsilon_m = 2, t_m = 1.49 \text{ mm}\)
#7: Inductive Square Loop
#8: \(\varepsilon_n = 2, t_n = 0.99 \text{ mm}\)
#9: \(\varepsilon_o = 4.50, t_o = 7.92 \text{ mm}\)

\[W = 13.87 \text{ mm}\]
\[d_1 = 0.99 \text{ mm}\]
\[d_2 = 0.49 \text{ mm}\]
\[T = 14.86 \text{ mm}\]
Fig. 6.33: Normal and 30° oblique (TE/TM) incidence transmission coefficient of the 2-layer inductive square loop FSS.

Fig. 6.34: Normal and 30° oblique (TE/TM) incidence transmission coefficient of the 7-layer dielectric material.
Fig. 6.35: Normal and 30° oblique (TE/TM) incidence transmission coefficient of the 9-layer dielectric/inductive square loop FSS.

6.11 Smart Surface Mushroom FSS

The mushroom band-gap structure proposed by Yablonovitch [60] is another novel design of the periodic structures. This structure has the potential to present an effective smart surface with the frequency variable reflection phase; and additionally it is able to suppress the surface waves successfully. The geometry of the structure, as depicted in Fig. 6.36, consists of a lattice of metallic patches connected to a metallic sheet by the vertical conducting vias. There is a dielectric material between the metallic plates.

The FDTD/Prony technique is applied to characterize the structure for the normal and oblique incidence TE waves. The reflection coefficient is equal to one similar to the
PEC ground plane. However, the reflection phase on the surface of the mushrooms, as observed in Fig. 6.37, has a frequency variation defining an effective surface with the smart property from PEC to PMC. This has the potential application for the low profile antenna designs, where the antenna can be brought very close to the 100% reflecting surface and still match the antenna and obtain a very good radiation characteristic.

The possibility of suppressing the surface waves utilizing the mushroom FSS structure is also explored in this work. In this manner, the performance of the 2 layers patch/via metallic elements shown in Fig. 6.38(a), for the normal and oblique incident TE waves propagating in the transverse plane ($k_z^i = 0$) is investigated. Notice that, in order to characterize the structure based on the developed FDTD technique, the geometry is assumed to be periodic in the $z$-direction. The grating PEC layers without the patches and vias (Fig. 6.38(b)) totally transmit the $z$-polarized incident waves, and no reflection occurs on the surface of the layered PEC. However, by adding the mushroom structure (Fig. 6.38(a)) an electromagnetic band-gap region for the surface waves is opened up which forbids the propagation of the EM waves in all directions in the transverse plane, as illustrated in Fig. 6.39.

Thus, the mushroom EBG, a metallo-dielectric band-gap structure, has the potential to generate a smart surface with the novel reflection phase properties from PEC to PMC. In addition, it can provide a complete surface wave band-gap region to effectively prevent the propagation of the EM waves in the transverse plane. These can be properly incorporated into the design of low profile antennas with the strongly reduced edge diffraction effects [60], [61].
Fig. 6.36: Mushroom EBG: (a) Periodic structure, (b) Unit cell of the structure. Note that the PBC walls are positioned on the PEC edges.

Fig. 6.37: Normal and oblique incidence reflection phase of the mushroom EBG structure. Note the phase variation from PEC to PMC. Reflection phase is computed on the surface of mushrooms.
Fig. 6.38: Suppression of the surface waves: (a) 2-Layer mushroom EBG, (b) PEC grating layers.

Note that the structures are periodic in the \( y-z \) directions.

Fig. 6.39: Normal and oblique incidence reflection coefficient of the 2-layer mushroom EBG structure.

Notice that the mushroom EBG opens up a complete surface waves band-gap region.
6.12 Summary

In this chapter, the FDTD/Prony technique is applied to characterize and obtain the propagation performance of some of the challenging and applicable frequency selective surfaces composed of arbitrary dielectric/metallic structures.

The broadband analysis of the periodic structures namely, layered dielectric materials, double concentric square loop FSS, dipole FSS manifesting high $Q$ resonance, square patch FSS manifesting high $Q$ resonance, crossed dipole FSS, dichroic plate FSS, inset self-similar aperture FSS, Sierpinski fractal FSS, layered tripod FSS, 9-layer dielectric/inductive square loop FSS, and smart surface mushroom FSS is successfully presented. The results are extremely satisfactory demonstrating the capability of the developed method in the broadband analysis of complex structures. Notice that the Prony’s scheme has been also applied to obtain the frequency response of the structures with much more accuracy and efficiency.

Also, in this work, the creation of the band-gap structures utilizing the cascaded FSS layers sandwiched between the dielectric slabs is investigated. Furthermore, the novel performance of the smart surface mushroom EBG, which has the potential applications in the communication antenna systems, is highlighted.
Chapter 7

Photonic Band-Gap Structures

Photonic band-gap structures are typically a class of periodic dielectric materials, which by generating an electromagnetic band-gap forbid the propagation of EM waves. The discovery of photonic band-gap materials has created unique opportunities for controlling the propagation of EM waves, leading to numerous novel applications in the optical and microwave technologies [62]-[68].

In this chapter, a comprehensive treatment of these potentially useful band-gap structures is provided. The objective is to characterize their interaction with the electromagnetic waves, identify their innovative propagation performances, incorporate the results into potential applications, and explore the possibility of modeling their characteristics using effective dielectric materials.

To accomplish this, the FDTD/Prony technique is applied to obtain and understand the band-gap behaviors of the triangular, rectangular, and woodpile PBG structures. The results are incorporated into three potential applications, namely, design of high $Q$ nanocavity lasers, guiding the EM waves in sharp bends, and miniaturized microstrip patch antennas. The effective dielectric material concept is also applied to investigate the possibility of modeling the performance of PBG using effective dielectric.
7.1 Band-Gap Characterization of PBG

The focus of this section is to present and study the band-gap characteristics of the PBG structures utilizing the FDTD/Prony technique. In this work, to obtain the performance of the PBG, the reflection coefficient of the plane wave incident on the band-gap structure is determined. Compared to the dispersion diagram method, which is usually applied to analyze these structures, the present technique appears to have two potential advantages:

- Obtaining reflection and/or transmission coefficients outside the band-gap regime,
- Presenting phase and polarization information of the scattered fields.

Obviously, using dispersion diagrams, it is not possible to obtain the reflection and transmission coefficients for the frequencies outside the band-gap. Additionally, phase and polarization of the reflected field cannot be deduced from a dispersion diagram. However, using the dispersion diagram one can effectively study the modal properties of the EM waves.

Notice that in the characterization of the periodic dielectric PBG structures, in order to keep the consistency with other chapters of this dissertation, the TE modes are defined as the waves where the electric field does not have any component in the direction normal to the surface of the periodic structure (x-direction); while the TM modes are the waves where the magnetic field does not have any component in the direction normal to the surface of the structure. These definitions are pictorially illustrated in Fig. 7.1 for a 2-D periodic triangular PBG structure of 5-layer infinite dielectric columns.
7.1.1 Triangular PBG

A 2-D periodic triangular PBG structure of 5-layer infinite dielectric columns inside the air region is depicted in Fig. 7.2. The structure is accurately analyzed using the FDTD technique, and the results for the normal and oblique incidence (in-plane angle) reflection coefficient are obtained in Fig. 7.3. As illustrated, there is a band-gap region for the waves polarized along the axis of the columns (TE band-gap) for the normalized frequencies $0.23 \leq a/\lambda_0 \leq 0.30$. The location of the band-gap region has an excellent agreement with the data presented in [12] based on the dispersion diagram.

The complementary geometry of Fig. 7.2 is presented in Fig. 7.4. The structure is a 5-layer PBG structure of air holes drilled in the dielectric material. The plane wave reflection coefficient is determined in Fig. 7.5. As shown, compared to the previous structure the connectivity in the high dielectric material is able to generate the TM band-gap (gap for the waves polarized in the transverse plane) in the normalized frequency range $0.20 \leq a/\lambda_0 \leq 0.26$.

The periodic contrast between the dielectric-air regions is the key for generating the band-gap. This contrast in the isolated (Fig. 7.2) or connected (Fig. 7.4) dielectric PBG results in the contrast of the energy confinement between the lower and upper band-gap edges for the TE or TM modes, respectively.

7.1.2 Rectangular PBG

The performance of a 2-D periodic rectangular PBG structure of 5-layer infinite dielectric columns inside the air region, as shown in Fig. 7.6, for the normal and oblique incidence
plane waves is determined in Fig. 7.7. As expected, a TE band-gap region for the normalized frequencies $0.22 \leq a/\lambda_0 \leq 0.32$ is achieved.

To obtain the TM band-gap region, a rectangular lattice of air holes drilled in the dielectric medium is presented in Fig. 7.8. The connectivity of the dielectric material opens up the gap regions for the TM waves determined in Fig. 7.9. It is demonstrated in Fig. 7.10 that by increasing the size of the holes the dielectric filling ratio of the PBG is reduced and the structure can be considered as the dielectric spots surrounded by the air holes. In this case, the near isolation of the dielectric columns is crucial to the production of gap region for the TE waves. The TE band-gap behavior of the structure for the frequency range $0.20 \leq a/\lambda_0 \leq 0.26$ is displayed in Fig. 7.11.

A rule of thumb may be developed that, the TE band-gaps are favored in a lattice of isolated high dielectric regions while the TM band-gaps are favored in a connected lattice. Arranging the PBG structure with both isolated spots and connected regions of dielectric material may results in the novel designs with complete band-gap regions [12].

7.1.3 Woodpile PBG

The woodpile PBG structure proposed by Ho et al. [69], is a novel design of the 3-D periodic dielectric materials with the complete band-gap region. The geometry of a 2-layer infinite square dielectric rods woodpile PBG is shown in Fig. 7.12. The structure is a composition of both separated and connected high dielectric regions in the three directions, which has the potential to generate a complete band-gap region and forbid the electromagnetic wave propagation in almost all directions.
The reflection coefficient for the normal, $30^\circ$ oblique/TE, and arbitrary incident plane wave $\theta^i = 40^\circ, \phi^i = 150^\circ$ with polarization angle $\psi^i = 45^\circ$ is presented in Fig. 7.13. As demonstrated, the structure is able to generate an almost complete band-gap region independent of the angle and polarization of the incident wave. By increasing the number of the periodic layers one can design a structure with the better band-gap performance.

The phase behavior of the reflected field on the surface of the woodpile for the $30^\circ$ oblique wave is investigated in Fig. 7.14. It is observed that, within the gap region the phase has an almost linear frequency variation. Thus, inside the band-gap, the woodpile structure may be modeled using an effective PEC plane where its location is variable (depend on the incident angle) instead to be fixed at the front surface of PBG. In some situations, the effective reflection plane concept is a valuable tool to approximate the performance of the PBG using the effective PEC plane [70].

### 7.1.4 Effective Dielectric Material

The objective in this section is to explore the possibility of modeling the behavior of the periodic dielectric PBG structure using the effective dielectric material. Based on the effective medium concept, the effective dielectric $\varepsilon_{\text{eff}}$ of the periodic composite structure of air/dielectric regions at long wavelengths can be approximated using the equation [71], [72]

$$
\varepsilon_{\text{eff}} = f_r \cdot \varepsilon_d + (1 - f_r) \cdot \varepsilon_a
$$

(7.1)
where \( f_r \) is the dielectric filling ratio, and \( \varepsilon_d \) and \( \varepsilon_a \) are the dielectric constants of the material and air \( (\varepsilon_a = 1) \) regions, respectively (Fig. 7.15).

The rectangular PBG structure of the isolated dielectric columns (Fig. 7.6, TE band-gap) is modeled using the one-dimensional periodic air/dielectric multi-layered structure in Fig. 7.16. The layered planar structure is analyzed based on the vector wave solution of Maxwell’s equations (Chapter 2). The band-gap characteristics of the triangular and rectangular PBG are compared with the effective dielectric model in Fig. 7.17, for the 60° oblique incidence wave. The periodic isolated dielectric layers of the effective model create a TE band-gap region illustrated in Fig. 7.17. For the small values of the lattice constant \( a \) the PBG and effective dielectric has almost the similar performance. However, by increasing the lattice constant and especially inside the gap region, the effective dielectric cannot model the PBG structure.

The behaviors of the triangular and rectangular PBG of the connected lattice are compared with the effective dielectric model (Fig. 7.18) in Fig. 7.19. In this case, the TM band-gap performance of the PBG is based on the connectivity in the high dielectric material; while the effective medium is a composition of the isolated dielectric layers and obviously it cannot model the characteristics of the PBG as illustrated in Fig. 7.19.
Fig. 7.1: Pictorial illustration of the (a) TE ($E_z$), and (b) TM ($H_z$) waves definitions for a 2-D triangular PBG structure including of 5-layer dielectric columns.
Fig. 7.2: 2-D triangular PBG structure including of 5-layer dielectric columns (finite in $x$ direction, infinite in $y$-$z$ directions): (a) Periodic structure, (b) Unit cell of the structure. Note that the PBC walls are positioned on the dielectric sides.

Fig. 7.3: Normal and oblique incidence reflection coefficient of the triangular PBG of dielectric columns. The isolated dielectric regions open up the TE ($E_x$) band-gap.
Fig. 7.4: 2-D triangular PBG structure including of 5-layer air holes (finite in $x$ direction, infinite in $y$-$z$ directions): (a) Periodic structure, (b) Unit cell of the structure. Note that the PBC walls are positioned on the dielectric sides.

Fig. 7.5: Normal and oblique incidence reflection coefficient of the triangular PBG of air holes. The connected dielectric regions open up the TM ($H_y$) band-gap.
Fig. 7.6: 2-D rectangular PBG structure including of 5-layer dielectric columns (finite in $x$ direction, infinite in $y$-$z$ directions): (a) Periodic structure, (b) Unit cell of the structure. Note that the PBC walls are positioned on the dielectric sides.

Fig. 7.7: Normal and oblique incidence reflection coefficient of the rectangular PBG of dielectric columns. The isolated dielectric regions open up the TE ($E_z$) band-gap.

$\varepsilon_r = 11.40$
$r = 0.28a$
$T_y = a, T_z = 0.20a$
Fig. 7.8: 2-D rectangular PBG structure including of 5-layer air holes (finite in $x$ direction, infinite in $y$-$z$ directions): (a) Periodic structure, (b) Unit cell of the structure. Note that the PBC walls are positioned on the dielectric sides.

Fig. 7.9: Normal and oblique incidence reflection coefficient of the rectangular PBG of air holes. The connected dielectric regions open up the TM ($H_z$) band-gap.

\[ \varepsilon_r = 11.40 \]
\[ r = 0.39a \]
\[ T_y = a, T_z = 0.20a \]
Fig. 7.10: 2-D rectangular PBG structure including of 5-layer large size air holes (finite in $x$ direction, infinite in $y$-$z$ directions): (a) Periodic structure, (b) Transverse plane representing the dielectric spots surrounded by the air holes, (c) Unit cell of the structure. Note that the PBC walls are positioned on the dielectric sides.

Fig. 7.11: Normal and oblique incidence reflection coefficient of the rectangular PBG of air holes. The large size of the air holes generates the isolated dielectric columns opening up the TE ($E_z$) band-gap.
Fig. 7.12: 2-Layer square dielectric rods woodpile PBG structure (finite in $x$ direction, infinite in $y$-$z$ directions): (a) Periodic structure, (b) Unit cell of the structure. Note that the PBC walls are positioned on the dielectric sides.

Proper arrangements of the isolated and connected dielectric regions open up an almost complete band-gap.

Fig. 7.13: Normal and oblique incidence reflection coefficient of the square dielectric rods woodpile PBG.

Proper arrangements of the isolated and connected dielectric regions open up an almost complete band-gap.
Fig. 7.14: 30° incidence reflection phase of the square dielectric rods woodpile PBG. Reflection phase is computed on the surface of structure. Notice that the phase has an almost linear frequency variation within the band-gap.

\[
f_r = \frac{\text{area of dielectric region inside the triangle}}{\text{area of triangle}}
\]

Fig. 7.15: Effective dielectric model for composite periodic structure of air/dielectric regions: (a) Periodic structure, (b) Effective dielectric material.
Fig. 7.16: Multi-layered one-dimensional periodic dielectric structure (finite in $x$ direction, infinite in $y$-$z$ directions). Effective dielectric model for the isolated columns rectangular PBG in Fig. 7.6.

\[ \varepsilon_i = 5.40, t_i = 0.55a \]
\[ \varepsilon_z = 1.00, t_z = 0.45a \]

Fig. 7.17: Comparative study between the performance of triangular and rectangular isolated columns PBG with the effective dielectric model for the $60^\circ$ oblique incidence TE wave ($E_z$).
Fig. 7.18: Multi-layered one-dimensional periodic dielectric structure (finite in $x$ direction, infinite in $y$-$z$ directions). Effective dielectric model for the connected lattice rectangular PBG in Fig. 7.8.

$\varepsilon_1 = 5.06, t_1 = 0.78a$

$\varepsilon_2 = 11.40, t_2 = 0.22a$

Fig. 7.19: Comparative study between the performance of triangular and rectangular connected lattice PBG with the effective dielectric model for the $60^\circ$ oblique incidence TM wave ($H_z$).

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7.2 Representative Applications of PBG

In this section, the novel characteristics of the PBG structure are integrated into the three representative applications, namely, high $Q$ nanocavity lasers, guiding the EM waves in sharp bends, and miniaturized microstrip patch antennas. A comparative study between the PBG and effective dielectric in the design of cavity, waveguide, and patch antenna is also presented. It is obtained that the performance of the PBG within the band-gap region cannot be modeled using the effective dielectric materials.

7.2.1 High $Q$ Nanocavity Lasers

Confinement of the electromagnetic waves in a very small volume has important consequences in the property of the optical emission such as reduction in size and power requirement of the integrated optical components, controlling spontaneous emission, and higher modulation speeds [73]-[76]. The goal of this section is to study a high $Q$ nanocavity laser using the PBG structure and compare its performance with the effective dielectric model.

A. PBG Cavity

To design a high $Q$ nanocavity laser a finite thickness 2-D PBG structure is used to localize the electromagnetic waves inside a defect region in three directions, based on the PBG gap/total internal reflections [75]. The defect-excited mode (inside the gap region) is localized in the transverse plane utilizing the PBG structure. In the vertical direction,
the dielectric contrast between the impurity and outside air region generates the total internal reflections as it traps the waves in this direction.

Fig. 7.20 shows a 2-D PBG structure including 3-layer photonic cells of the triangular lattice of air holes drilled in a dielectric host medium with $\varepsilon_h = 11.56$ (which corresponds to GaAs) and slab thickness $d = 0.57a$. The air holes have the radius $r = 0.29a$. This structure generates a TM band-gap for the electric field polarized in the plane of periodicity. The center hole of the PBG is removed to generate a defect region and is excited by a $y$-directed infinitesimal dipole. The $y$-dipole mode cannot propagate through the PBG in the transverse plane and in the vertical direction it is trapped by the way of total internal reflection at the dielectric-air interfaces. The EM fields are localized inside the defect and at the resonance frequency generate a high $Q$ dielectric nanocavity laser.

The characterization of the finite size PBG structure is obtained based on the FDTD technique. The resonance frequency is determined at $f_0 = 128 \text{THz}$ (infrared), where $a/\lambda_0 = 0.29$ (notice that the mid-gap frequency for a finite thickness PBG compared to the infinite one is increased [75]). Figs. 7.21(a) and 7.21(b) show the near field patterns of the PBG structure in the transverse and vertical planes, respectively. As observed the EM waves are trapped in the defect region by the periodic PBG /total internal reflection.
B. Effective Dielectric Cavity

To investigate the possibility of modeling the PBG in confining the EM waves using effective medium concept, the periodic structure is replaced by an effective dielectric (Eq. (7.1)), as shown in Fig. 7.22. Two different cases are followed to present the comparative study between the PBG and effective dielectric:

Case I (inside the band-gap): In this case the PBG structure, operating within the gap region, with 31% air filling ratio is replaced by an effective dielectric $\varepsilon_{\text{eff}} = 8.29$ (the defect dielectric constant is the same as before, $\varepsilon_{\text{def}} = 11.56$). The near field patterns of the effective dielectric in the transverse and vertical planes are plotted in Figs. 7.23(a) and 7.23(b). As observed, compared to the PBG’s patterns (Fig. 7.21), in the transverse plane the EM waves propagate outside the defect region without any strong reflection on the boundary of the cavity; however, in the vertical plane the waves are trapped similar to the PBG. To design a high $Q$ cavity, one needs a strong reflection on the circular wall of the cavity to localize the EM waves in the transverse plane. Therefore, in this case the effective dielectric cannot properly model the PBG for localizing the EM waves.

Case II (outside the band-gap): In the second case the performance of the PBG is compared with the effective dielectric model for the normalized frequencies outside the band-gap. A PBG structure with $d = 0.66a$, $r = 0.33a$, and $a = 0.12\lambda_0$ is designed to operate at $f_0 = 124THz$ outside the gap region. The structure with 40% air filling ratio
is modeled using the effective material $\varepsilon_{\text{eff}} = 7.34$. Figs. 7.24(a) and 7.24(b) display the field patterns in the transverse plane for both PBG and effective dielectric structures. As observed, since the PBG operates outside the gap region, both structures have almost the same behaviors and they cannot localize the EM waves.

It is concluded that for the frequencies within the band-gap, where the lattice constant is in the order of dielectric wavelength, the PBG cannot be modeled using the effective dielectric; however, outside the band-gap with small lattice constants (compared to the dielectric wavelength), the PBG and effective dielectric have almost similar behaviors.

The concept of PBG in localizing the EM waves is based on creation a band-gap with an almost 100% reflection in all directions, and then the defect excited modes within the band-gap region in not able to propagate through the PBG. But, the effective dielectric medium controls the propagation of the EM waves based on total internal reflection, which it cannot be occurred in all directions on the circular boundary of the cavity. To increase the trapping of the EM waves using the effective dielectric medium, one can increase the contrast between the defect and effective material. Fig. 7.25 shows the field pattern of the effective dielectric $\varepsilon_{\text{eff}} = 8.29$ with relatively large defect region $\varepsilon_{\text{def}} = 49$, and as observed the waves are almost confined inside the cavity.

The electric and magnetic energy densities $W_e$ and $W_m$ inside the cavity region can be determined by the following equations

$$W_e = \frac{\varepsilon}{2} |E|^2$$  \hspace{1cm} (7.2a)
\[ W_m = \frac{\mu}{2} |\mathbf{H}|^2. \]  

(7.2b)

Using these equations the total stored energy inside the defect region for three different cases, PBG/inside the band-gap and effective material \( \varepsilon_{\text{eff}} = 8.29 \) with two different defect dielectrics \( \varepsilon_{\text{def}} = 11.56 \) and \( \varepsilon_{\text{def}} = 49 \) is computed in Fig. 7.26. The slope of the exponential decay of the energy can be used to obtain the quality factor (\( Q \)) of the cavity. Note that, this method is useful where the \( Q \) of the system is not very large. It is shown that the PBG structure generates the highest quality factor with \( Q = 116 \).

\[ \varepsilon_{\text{def}} = 11.56 \quad \varepsilon_{\text{def}} = 49 \]

\[ \text{C. 7-Layer High } Q \text{ PBG Structure} \]

As mentioned earlier, the \( Q \) of the PBG is controlled by the periodic structure in the transverse direction (\( Q_\parallel \)) and total internal reflection in the vertical direction (\( Q_\perp \)). The total \( Q \) is calculated using the following equation:

\[ \frac{1}{Q} = \frac{1}{Q_\parallel} + \frac{1}{Q_\perp}. \]  

(7.3)

The \( Q_\parallel \) is increased by using more number of photonic cells, and the \( Q_\perp \) is controlled by changing the defect dielectric constant as it controls the trapping of the EM waves in the vertical direction. To design a high \( Q \) nanocavity laser both \( Q_\parallel \) and \( Q_\perp \) should have the large values.

Fig. 7.27 depicts a 7-layer photonic cells PBG structure with \( d = 0.50a \) and \( r = 0.31a \). Total stored energy inside the defect region is obtained in Fig. 7.28. In the first case a defect region with the same dielectric constant as the host medium
(ε_{def} = ε_h = 11.56) is used. Since in this design the number of photonic cells is increased to 7 layers, the quality factor is also increased to about Q = 210 at the normalized frequency \( a/\lambda_0 = 0.30 \) (\( f_0 = 125 THz \)).

By varying the defect dielectric constant, both defect band frequency and total Q of the structure are changed, as determined in Figs. 7.29 and 7.30. It is obtained that at the normalized frequency \( a/\lambda_0 = 0.346 \) (\( ε_{def} = 4.0 \)), a relatively high Q structure is designed and the modes are strongly localized and cannot be leaked to the outside region. The quality factor for this excited defect band (\( a/\lambda_0 = 0.346, f_0 = 140 THz \)) is about \( Q = 1050 \).
Fig. 7.20: Cavity design using finite thickness 2-D triangular PBG structure: (a) 3-D geometry, (b) Cross section of the PBG ($x$-$y$ plane).

Fig. 7.21: Near field patterns of the PBG: (a) Transverse plane, (b) Vertical plane.

Fig. 7.22: Effective dielectric model for the PBG periodic structure.
Fig. 7.23: Near field patterns of the effective dielectric as it models the PBG (inside the band-gap):
(a) Transverse plane, (b) Vertical plane. Note the lack of similarity between the field patterns of effective dielectric and PBG (Fig. 7.21(a)) in transverse plane.

Fig. 7.24: Near field patterns of the (a) PBG (outside the band-gap) compared to the (b) effective dielectric. Note the similarity between the field patterns.
Fig. 7.25: Near field pattern of the effective dielectric material in the transverse plane with a relatively high dielectric defect region. The waves are almost confined inside the cavity.

Fig. 7.26: Total stored energy inside the defect region for the PBG compared to the effective dielectric materials. Note that the PBG has the highest quality factor.
Fig. 7.27: High $Q$ nanocavity laser utilizing 7-layer PBG structure.

Fig. 7.28: Total stored energy inside the defect region of the 7-layer PBG for the same and different defect dielectrics (compared to the host medium).
Fig. 7.29: Defect band frequency of the 7-layer PBG for various defect dielectrics.

Fig. 7.30: Total $Q$ of the 7-layer PBG versus normalized frequency (varying defect dielectric).

Notice that the maximum $Q$ ($Q = 1050$) occurs around the normalized frequency $a/\bar{\lambda} = 0.346$ ($\varepsilon_{def} = 4.0$).
7.2.2 Guiding the EM Waves in Sharp Bends: Channelizing the Light

The efficient guiding of the electromagnetic waves in the straight and sharp bends is an important consideration for telecommunication and optical system applications [77]. The conventional waveguides can support the guided modes in the straight channels with high efficiency yet are restricted to the radiation loss at the bending radius. Photonic band-gap structures have the potential application to guide the modes either along a straight line or around a sharp corner with a great efficiency [78]-[80]. The focus of this section is to design a finite thickness 2-D PBG structure to guide the EM waves at 90° and 60° bends and present a comparative study with the effective dielectric material.

A. Guiding the Waves at 90° and 60° Bends utilizing PBG

To design a PBG structure for guiding the EM waves, an array of the dielectric rods or air holes in the PBG material is removed to present a channel in the guiding direction. The modes with their frequencies within the gap-region are confined through the channel, and they cannot be scattered through the PBG, even at tight corners. In this work a triangular PBG structure of air holes drilled in the dielectric material is used to guide the EM waves in sharp corners.

Fig. 7.31(a) depicts the geometry of a PBG structure, which is designed to provide the guiding of EM waves at the 90° bend (an array of the holes in 90° channel is removed). The air holes have the radius \( r = 0.36a \), and are drilled in the dielectric medium with \( \varepsilon_h = 11.56 \) and thickness \( d = 0.57a \). The PBG with the TM band-gap (electric field inside the transverse plane) characteristics effectively supports the guided
mode with the normalized frequency $a/\lambda_0 = 0.32 \ (f_0 = 137\, THz)$. Notice that, it is obtained that within the band-gap frequency region, the excited mode with the frequency $f_0 = 137\, THz$ has the best guiding performance through the channel. The guided mode is excited by a $y$-directed infinitesimal dipole in the left side of the channel. The near field pattern in the transverse plane is plotted in Fig. 7.31(b). As illustrated, the PBG structure guides the EM waves inside the $90^\circ$ channel with a high efficiency. The generated band-gap of the periodic structure confines the mode inside the channel even at the sharp corner. In the vertical direction, as discussed in Sec. 7.2.1, the total internal reflection traps the waves inside the slab.

Guiding the waves at the $60^\circ$ corners utilizing the PBG structures is also studied in this work. To this end, an array of the PBG holes in the $60^\circ$ bend is removed to provide the guiding direction for the $60^\circ$ channel, as illustrated in Fig. 7.32(a). The PBG is made of the air holes with radius $r = 0.44a$ in the host medium with $\varepsilon_h = 11.56$ and thickness $d = 0.44a$. In this case, the holes have the large size radius to allow the opening up the TE ($E_z$) band-gap as well as the TM ($H_z$) one, as explained in Sec. 7.1. This improves the gap phenomena of the structure. It is determined that the excited mode at the normalized frequency $a/\lambda_0 = 0.42 \ (f_0 = 140\, THz)$, inside the band-gap, can be effectively guided through the channel. The near field pattern at this normalized frequency is plotted in Fig. 7.32(b). As noticed, the EM waves are slightly scattered through the PBG at the tight turn, and they are not channeled into the $60^\circ$ bend as well as
the 90° case. This could be because of the difference in the shape of the turns in 60° and 90° channels.

Transmission through the bend can be visualized as a scattering process [78], where the mode propagating in the first branch of the channel with the wave vector \( \mathbf{k}_1 \) is scattered into the bend with the wave vector \( \mathbf{k}_s \), and then propagates through the second branch with the wave vector \( \mathbf{k}_2 \). Shaping the bend has an advantage to increase the coupling of the EM waves between the two branches of the channel. To improve the guiding of the propagated mode inside the 60° bend, two small holes with radius \( r_0 = 0.17a \) are introduce at the corner of the channel, as demonstrated in Fig. 7.33(a). The near field pattern of the 60° shaped bend is plotted in Fig. 7.33(b). As observed, the small holes have the potential to properly shape the bend in order to reduce the scattering of the EM waves through the PBG and improve the coupling of the waves at the sharp corner. A big hole at the 60° turn can also effectively bend the light in the corner, illustrated in Fig. 7.34.

**B. Guiding at the Sharp Corners based on the Effective Dielectric**

This section addresses the performance of the effective dielectric materials for guiding the EM waves at the tight turns compared to the PBG structures. The PBG structure for guiding in the 90° channel (Fig. 7.31(a)) with 47% air filling ratio is modeled using the effective dielectric with \( \varepsilon_{eff} = 6.60 \) in Fig. 7.35(a). The dielectric constant of the channel is the same as before (\( \varepsilon_h = 11.56 \)).
It is observed in Fig. 7.35(b) that the total internal reflection guides the excited mode inside the straight channel effectively. However, at the 90° corner, the condition for the total internal reflection cannot be satisfied and the EM waves are scattered through the effective material. Notice that the band-gap phenomenon of the PBG is based on the periodicity of the structure, which it can be successfully applied at the sharp corners.
Fig. 7.31: PBG for guiding the waves in 90° bend: (a) Cross section of the structure, (b) Near field pattern.

Fig. 7.32: PBG for guiding the waves in 60° bend: (a) Cross section of the structure, (b) Near field pattern.
Fig. 7.33: PBG for guiding the waves in 60° shaped bend utilizing two small holes: (a) Cross section of the structure, (b) Near field pattern. Notice that the small holes extremely improve the coupling of the waves in the corner (compared to the Fig. 7.32(b)).

Fig. 7.34: PBG for guiding the waves in 60° shaped bend utilizing one big hole: (a) Cross section of the structure, (b) Near field pattern. Notice that the structure has almost the similar performance as the structure in Fig. 7.33(a).
Fig. 7.35: Effective dielectric for guiding the waves in 90° bend: (a) Cross section of the structure, (b) Near field pattern. Notice that the structure cannot turn the waves similar to the PBG (Fig. 7.31(b)).

7.2.3 Miniaturized Microstrip Patch Antennas

Smaller physical size and wider bandwidth are two engineering aspects of great interest in the wireless world. Microstrip patch antennas on the thick and high dielectric substrates provide desired features, namely, small element size and large bandwidth. However, this type of substrates increases the surface waves resulting in the unwanted interference pattern of edge diffraction/scattered and leaky waves. The main objective of this section is to answer the key engineering question that: how to suppress the surface waves and design a high performance miniaturized patch antenna with relatively
improved bandwidth? In this manner, the performance of the dielectric PBG and effective materials in suppressing the surface waves are investigated. Potential applications of the PBG/effective dielectric as the substrate for the miniaturized patch antennas with reduced surface waves are presented.

A. Microstrip Patch Antenna

Microstrip patch antennas are the fascinating low profile planar structures that have a very wide range of applications due to their simplicity in both operation and fabrication, and their conformability and integrability with the planar circuitry. Fabricating patch antennas on a high dielectric constant substrate has the benefit of reducing the element size of the patch radiator. Besides reducing the element size, being able to integrate the patch antenna directly on the Monolithic Microwave Integrated Circuit (MMIC) simplifies the interconnection of the antenna with circuitry.

However, the high dielectric constant substrate increases the trapping of the substrate modes and reduces the bandwidth of the patch. To increase the bandwidth, a thicker substrate may be used, but it increases the amount of the energy trapped in the substrate and causes pattern degradation from the diffraction/scattering of the substrate modes at the finite substrate’s edges [81].

Fig. 7.36 depicts the geometry of a patch antenna with 12\text{mm} width and 16\text{mm} length on a finite substrate with dielectric constant $\varepsilon_r = 10.20$. To compare the performance of the conventional and PBG substrates, in this work a relatively large size substrate (to design a high performance dielectric PBG with enough periodic layers) with
the dimension around $3\lambda_0$ (400mm) is used. The FDTD technique is applied to obtain the radiation characteristics of the patch radiator on the dielectric substrate for two different thicknesses $t = 4\ mm$ and $t = 8\ mm$. As determined in Fig. 7.37, the thinner substrate has the resonance frequency at $f_0 = 2.42\ GHz$ with the 10-dB bandwidth $BW = 1.4\%$. Doubling the thickness of the substrate increases the bandwidth to about 4.3%. The resonance frequency is around $f_0 = 2.20\ GHz$. Notice that for both thin and thick substrates, the resonance frequency is below the cutoff frequency of the TE$_{z1}$ mode and only the TM$_{z0}$ ($E_z$) exists supporting the surface waves.

Near and far field patterns of the thin and thick substrates are presented in Figs. 7.38 and 7.39. For both cases the electrical size of the substrate is kept fix around $3\lambda_0$ to reveal the effect of the edge diffraction/scattered surface waves on the radiation patterns of the patch radiator. Comparing the radiation performance of two substrates, illustrates the development of more surface waves for the thicker one. This results in the directivity ripples and increased back radiation of about $-4.5\ dB$.

In the following sections the design of a miniaturized patch antenna with significantly reduced surface waves is explored. The substrate beneath the patch is the high/thick dielectric material ($\varepsilon_r = 10.20, t = 8\ mm$) to achieve the similar resonance performance as the conventional thick substrate [82]. Around this substrate a PBG or effective dielectric is used to prevent the propagation of the surface waves.
B. PBG Substrate

The focus of this section is to address a miniaturized patch antenna using the PBG substrate with suppressed surface waves. The substrate modes are mostly polarized along the z-axis, and as illustrated in Sec. 7.1.2, the dielectric PBG including of large size air holes has the potential to open up the gap region for the z-polarized waves. Fig. 7.40 depicts the geometry of a rectangular PBG of 5-layer infinite air holes with $r/a = 0.45$ drilled in the dielectric material with $\varepsilon_r = 10.20$. The FDTD/Prony technique is applied to characterize and present the normal and oblique incident plane wave reflection coefficient of the structure. As observed in Fig. 7.41, there is a band-gap region for the TM$_z$ waves for the normalized frequencies $0.21 \leq a/\lambda_0 \leq 0.28$ (or $0.67 \leq a/\lambda \leq 0.89$ in terms of the dielectric wavelength $\lambda$).

However, for suppression the surface waves, the PBG with a finite thickness is used as the substrate around the patch, and one needs to understand the performance of the finite thickness dielectric PBG on the metallic plate. To this end, the PBG structure with thickness $t = 0.20a$ presented in Fig. 7.42(a) is characterized utilizing the FDTD technique. In this study, it is assumed that the structure is periodic in the z-direction in order to apply the FDTD technique with the periodic walls. The grating PEC layers without the PBG material (Fig. 7.42(b)) totally transmit the incident waves and no reflection occurs on the surface of the structure. By adding the PBG material (Fig. 7.42(a)) a band-gap region is opened up which it prevents the propagation of the EM waves, as seen in Fig. 7.43. It is noted that, by changing the plane wave incident angles
the gap region is shifted and the structure cannot generate any complete surface wave band-gap [83].

Fig. 7.44 depicts a PBG structure with less than 4-layer photonic cells and normalized frequency $a/\lambda_o = 0.35$ surrounding the high dielectric material beneath the patch. The thickness is $t = 8\, \text{mm}$ and the size of the structure is around $3\, \lambda_o$. The near and far field characteristics are presented in Fig. 7.45. As observed, although the PBG cannot reduce the surface waves in all directions, but still compared to the conventional thick substrate it has the great advantage to suppress the surface waves and improve the radiation pattern. The ripples in the antenna patterns are disappeared, and the back radiation is remarkably reduced to about $-8.5\, \text{dB}$. The directivity is around $5\, \text{dB}$. The resonance performance is almost the same as the conventional thick substrate, shown in Fig. 7.37.

Notice that, as illustrated in the previous chapter (Sec. 6.11), the metallo-dielectric mushroom EBG has the potential to produce the complete surface waves band-gap region. Therefore, the mushroom EBG may be successfully integrated as the substrate around the patch to design a miniaturized antenna with the higher radiation performance and much more reduced edge diffraction effects, compared to the dielectric PBG.

C. Effective Dielectric Substrate

This section explores the possibility of modeling the dielectric PBG structure in suppressing the surface waves, using the effective dielectric material. The idea is to lower
the dielectric constant of the material surrounding the patch antenna while not affecting the substrate beneath the patch [82]. Drilling an array of the small holes on the substrate around the patch can generate a perforated substrate with less effective dielectric constant. The dielectric contrast between the materials beneath and around the patch provides the total internal reflections inside the region under the path and reduces the propagation of the surface waves through the substrate.

In this study, based on the size of the holes drilled in the PBG and using the effective dielectric medium concept (Eq. (7.1)), an effective substrate material with $\epsilon_r = 4.30$ is used around the patch, shown in Fig. 7.46. The near and far fields are obtained in Fig. 7.47. As observed, compared to the conventional thick substrate, the surface waves are reduced, and the radiation patterns are noticeably improved. The directivity is $4.2 \, dB$ and is $0.8 \, dB$ less than the PBG structure, and the back radiation is around $-7.4 \, dB$ that is $1.1 \, dB$ higher than the PBG. The similar resonance behavior as the conventional thick substrate is determined in Fig. 7.37.

Note that, both dielectric PBG and effective dielectric cannot suppress the surface waves in all directions and they have almost the similar performance in reducing the surface waves.
Fig. 7.36: Patch antenna on the conventional substrate. Note to the edge diffraction/scattered and space waves.

Fig. 7.37: Return loss ($S_{11}$) of the patch antenna on the conventional thin, thick, PBG, and effective dielectric substrates.
Fig. 7.38: Performance of the patch antenna on the conventional thin substrate, (a) Near field pattern, (b) Far field patterns.

Fig. 7.39: Performance of the patch antenna on the conventional thick substrate, (a) Near field pattern, (b) Far field patterns. Note the pattern bifraction near $\theta = 0^\circ$ and high back radiation.
Fig. 7.40: 5-Layer photonic cells rectangular PBG of infinite air holes.

Fig. 7.41: Band-gap region of the dielectric PBG for the waves propagating through the PBG.
Fig. 7.42: Exploring the possibility of suppression the surface waves: (a) 5-Layer finite thickness PBG, (b) PEC grating layers. Note that the structures are periodic in the y-z directions.

Fig. 7.43: Band-gap region of the dielectric PBG for the surface waves propagating in the transverse plane. Note that the PBG is not able to generate any complete surface wave band-gap.
Fig. 7.44: Patch antenna on the PBG substrate: (a) 3-D geometry, (b) Cross section of the PBG (x-y plane).

Fig. 7.45: Performance of the patch antenna on the PBG substrate, (a) Near field pattern, (b) Far field patterns.

Far field patterns.
Fig. 7.46: Patch antenna on the effective dielectric substrate.

Fig. 7.47: Performance of the patch antenna on the effective dielectric substrate, (a) Near field pattern, (b) Far field patterns.
7.3 Summary

The main objective of this chapter is to present the electromagnetic performance of the periodic dielectric PBG structures and incorporate their unique propagation characteristics into some potential applications as high $Q$ nanocavity lasers, guiding the EM waves at sharp bends, and miniaturized microstrip patch antennas.

The FDTD/Prony technique is successfully applied to obtain the broadband characteristics of the triangular, rectangular, and woodpile PBG structures. It is observed that the isolated dielectric regions create the TE ($E_z$) band-gap while the connected dielectric lattice produces the TM ($H_z$) band-gap. A 3-D periodic composite material of both isolated dielectrics and connected lattice has the potential to generate a complete band-gap structure for all angles of arrival and for all polarization states, as appeared in the novel woodpile PBG structure.

The possibility of modeling the dielectric PBG structures using the effective dielectric media is also investigated. It is shown that, the effective periodic layered dielectric material cannot model the performance of the PBG for large lattice constant within the band-gap. For small lattice constants, outside the gap region, PBG and effective medium have almost the similar behaviors.

A finite thickness 2-D PBG structure is presented to localize the EM waves in three directions and design a high $Q$ nanocavity laser. The periodic PBG structure forbids the propagation of the defect excited mode (inside the band-gap) in the transverse plane, and the total internal reflection traps the EM waves in the vertical direction. It is observed that using a proper number of photonic cells and a suitable defect dielectric constant a
nanocavity laser with a relatively high quality factor \( Q \) can be designed. Also, it is illustrated that the effective dielectric medium cannot model performance of the PBG in designing the high \( Q \) cavity laser.

Potential application of the PBG structure for guiding the EM waves at sharp bends namely, 90° and 60° corners is highlighted. To this end, an array of the PBG holes in the guiding direction is removed. The guiding modes with their frequencies within the gap-region are confined inside the channel, and they cannot be scattered through the PBG. Additionally, a novel shaped bend, constructed using small holes at the corner, is introduced to improve the coupling of the EM waves in the channels. Furthermore, it is shown that the effective material is not able to model the PBG in turning the waves.

Designing the miniaturized microstrip patch antenna on the thick and high dielectric substrate utilizing the PBG and effective dielectric materials for suppression of the surface waves is also explored. It is demonstrated that although both dielectric PBG and effective material cannot suppress the surface waves in all directions, but still they have the advantage to reduce the surface waves and considerably improve the radiation patterns of the miniaturized antenna. The mushroom EBG structure with the complete surface wave band-gap region can be potentially useful to prevent the propagation of the surface waves in all directions and design the miniaturized antenna with the higher radiation characteristics and much more reduced edge diffraction effects.
Chapter 8

Composite Media with Negative Permittivity and Permeability Properties

The objective in this chapter is to characterize a novel structural configuration composed of periodic array of conducting straight wires/Split Ring Resonators (SRR) that exhibits a frequency range with simultaneously negative values of effective permittivity and permeability [13]. This new class of material is called the Left-Handed (LH) medium and has some unusual electromagnetic properties such as reversal of the Doppler shift, anomalous refraction, and reversal of radiation pressure to radiation tension. These behaviors can never be observed in naturally occurring materials or composites.

The FDTD/Prony technique is successfully applied to determine the characteristics of this electromagnetic periodic structure. It is shown that a pass-band occurs within the gap region of the composite medium where both permittivity and permeability are negative; which it demonstrates the existence of the LH material. The analysis methodology based on the FDTD technique is a capable engine to understand the unique propagation behaviors of the LH media. The obtained results can be properly integrated into some future potential applications of the composite LH materials.
8.1 Electromagnetic Properties of LH Materials

The concept of the LH materials is introduced by Veselago [84] in 1968. He theoretically investigated the electrodynamics consequences of the medium having both negative permittivity and permeability. Applying Maxwell’s equations in a LH material, the electromagnetic plane waves are represented as

\[ E = E_0 e^{-jkr} \]  
\[ H = H_0 e^{-jkr} \]  

where \( k = \hat{k} k \), \( k = \omega \sqrt{\mu_{\text{eff}}(\omega)\varepsilon_{\text{eff}}(\omega)} \) (\( \mu_{\text{eff}} \) and \( \varepsilon_{\text{eff}} \) are the effective permeability and permittivity of the material), and

\[ H_0 = \frac{k}{\omega \mu_{\text{eff}}(\omega)} \hat{k} \times E_0. \]  

It is determined from Eq. (8.2) that in the materials with negative effective permittivity and permeability, \( E, H, \) and \( k \) form a left-handed triplet of vectors (\( \mu_{\text{eff}}(\omega) \) has the negative sign in Eq. (8.2)). Thus, in the LH materials the energy flux \( S = \frac{1}{2} \text{Re}[E \times H^*] \) is in the \( -\hat{k} \) direction, and it flows in a direction opposite to the plane wave propagation. Therefore, this new class of materials would have dramatically different propagation characteristics compared to the ordinary or Right-Handed (RH) materials. Some of the unique and potentially useful electrodynamics phenomena of the LH materials such as (a) reversal of the Doppler shift, (b) anomalous refraction, and (c) reversal of radiation pressure to radiation tension are briefed in the followings.
8.1.1 Reversal of the Doppler Shift

The group and phase velocities of the EM waves in an arbitrary medium are determined using the energy flux and wave vectors, respectively. Thus, in a LH material the group and phase velocities are in the opposite directions. Doppler shift in the RH/LH materials for a detector moving with the velocity $\mathbf{v}$ in the presence of the source with the frequency $\omega_0$ can be obtained as

$$\omega = \omega_0 \left(1 - \frac{\hat{s} \cdot \mathbf{v}}{v_g}\right)$$  \hspace{1cm} (8.3)

where $\hat{s}$ is the direction of the energy flux, $v_g$ is the group velocity, and $\omega$ is the receiving frequency. The sign change of $\hat{s} \cdot \mathbf{v}$ in the right-/left-handed media, reveals the reversal of the Doppler shift in the LH materials compared to the ordinary media.

8.1.2 Anomalous Refraction

Reflection and refraction behaviors of the EM waves at the boundary of the right/left-handed media are demonstrated in this section. As mentioned earlier, in a right-handed material both the wave vector $\mathbf{k}$ and energy flow $\mathbf{S}$ are in the same directions, while in the left-handed material they are in the opposite directions. In addition, from the boundary conditions between two different media 1 and 2

$$\hat{n} \times \mathbf{E}_1 = \hat{n} \times \mathbf{E}_2$$  \hspace{1cm} (8.4a)

$$\hat{n} \times \mathbf{H}_1 = \hat{n} \times \mathbf{H}_2$$  \hspace{1cm} (8.4b)

$$\varepsilon_1 \hat{n} \cdot \mathbf{E}_1 = \varepsilon_2 \hat{n} \cdot \mathbf{E}_2$$  \hspace{1cm} (8.4c)
\[ \mu_1 \hat{n} \cdot \mathbf{H}_1 = \mu_2 \hat{n} \cdot \mathbf{H}_2, \]  

(8.4d)

it is readily obtained that, on the boundary between the right/left-handed media, the EM fields have the same tangential components, while they have different normal components in the opposite directions.

The above phenomena are applied to obtain the wave vector \( \mathbf{k} \), energy flux \( \mathbf{S} \), and the \( \mathbf{E} \) and \( \mathbf{H} \) components of the incident, reflected, and refracted (transmitted) waves in the RH and LH materials, as illustrated in Fig. 8.1.

![Fig. 8.1: Reflection and refraction of the EM waves on the boundary between the right/left-handed materials.](image)

As observed, the refracted wave in the left-handed medium has the anomalous behavior and it undergoes with the angle \( \phi \left( k_1 \sin \theta = k_2 \sin \phi \right) \) with respect to the normal to the boundary. The reflected wave is inside the right-handed medium and it has the similar phenomena as the ordinary cases.
One of the useful applications of the above interesting phenomenon is presented in Fig. 8.2. A flat slab LH material with large enough thickness and proper negative permittivity and permeability has the potential to focus the point source radiation without any reflection (parameters of the medium can be selected in a way to obtain the zero reflection). This remarkably useful performance can never be achieved using the ordinary materials.

8.1.3 Reversal of Radiation Pressure to Radiation Tension

Another unusual behavior of the LH materials is reversal of the radiation pressure to radiation tension. The electromagnetic momentum \( \mathbf{g} \) related to the plane wave propagating in the \( \mathbf{k} \) direction is determined [84] as

\[
\mathbf{g} = \frac{W}{\omega} \cdot \mathbf{k}
\]

where \( \omega \) is the frequency and \( W \) is the total stored energy. As observed, in a right-handed material there is a pressure in the direction of propagation. However, in a left-handed material provided it is thick enough.

Fig. 8.2: The point source radiation, (a) is diverged by the flat slab RH material, and (b) is focused by the flat slab LH material provided it is thick enough.
material the vector \( \mathbf{k} \) is directed toward the source of radiation and instead of pressure a radiation tension or attraction is introduced.

These are some of the interesting features of the electrodynamic properties of the LH materials. The task in the following section is to introduce a composite structural configuration of conducting periodic elements with simultaneously negative permittivity and permeability characteristics.

### 8.2 Characterization of LH Materials

In this section, a periodic array of conducting wires with negative permittivity is properly combined with the split ring resonators with negative permeability to present the composite material with simultaneously negative permittivity/permeability [13]. To understand the physical behavior of the composite structure, the FDTD/Prony technique is applied to characterize the straight wires, SRR, and straight wires/SRR. Fig. 8.3 pictorially represents the electromagnetic performances of the media having negative effective permittivity, negative effective permeability, and both negative effective permittivity/permeability, based on the definition of \( \mathbf{k} \) vector \( (\mathbf{k} = \hat{k} \mathbf{k} = \hat{k} \mathbf{k}, k = \omega \sqrt{\mu_{\text{eff}}(\omega)\varepsilon_{\text{eff}}(\omega)} ) \).

#### 8.2.1 Conducting Straight Wires

A 2-layer periodic array of conducting straight wires is presented in Fig. 8.4. The reflected power for the normal and 30° oblique \( (\theta^i = 90°, \phi^i = 150°) \) incident plane
waves $E_z$ (TE case) is determined in Fig. 8.5. As observed, there is a single gap in the propagation up to a cutoff frequency for modes with the electric field polarized along the axis of the wires. The gap corresponds to a region where either $\varepsilon_{\text{eff}}(\omega)$ or $\mu_{\text{eff}}(\omega)$ is negative (decaying modes) (Fig. 8.3). Since the straight wires are excited by the electric field, they cannot generate any effective permeability and the existence of the gap indicates the presence of a negative effective permittivity.

### 8.2.2 Split Ring Resonators

The periodic array of split ring resonators has been recently introduced by Pendry et al. [85] to extend the range of the electromagnetic properties of the effective media, and present a novel material with effective magnetic permeability. It gives one the opportunity to make a material with the negative permeability, from which a left-handed medium can be constructed.

There are two incident polarizations of interest: magnetic field dominantly polarized (a) along $(H_\parallel)$ and (b) perpendicular $(H_\perp)$ to the axes of the rings. In both cases the electric field is in the plane of the rings. Fig. 8.6 depicts the geometry of a 2-layer periodic array of SRR for the $H_\parallel$ case. The applied plane wave induces the currents that depending on the resonant properties of the structure produces a magnetic field, which may either oppose or enhance the incident field. In particular, since the array of the SRR is excited by the magnetic field polarized along the axes of the rings, the structure has the potential to generate the effective permeability $\mu_{\text{eff}}(\omega)$. The purpose of splitting
the rings is to generate a large capacitance in the small gap region between the rings in order to concentrate the electric field and lower the resonance frequency.

The reflected power for both cases of normal and $30^\circ$ oblique ($\theta^i = 90^\circ, \phi^i = 150^\circ$) plane waves (TE case) is presented in Fig. 8.7. The obtained magnetic gap regions are based on the negative effective permeability of the material. The performance of the SRR for the $H_\perp$ case (Fig. 8.8) is also studied, and the incident reflected power is determined in Fig. 8.9. In this case, the magnetic effects are small, and the $\mu_{\text{eff}}(\omega)$ is small, positive, and slowly varying. The negative effective permittivity of the structure generates the dielectric gap regions.

### 8.2.3 Conducting Straight Wires/Split Ring Resonators

To present a LH material, the straight wires are combined with the SRR for the $H_\parallel$ case, as shown in Fig. 8.10. The characteristics of the composite structure for the normal and oblique incident waves are presented in Fig. 8.11. It is clearly illustrated in Fig. 8.12 that around the frequency $5.3\, GHz$, a pass-band occurs within the previously forbidden gaps of both the conducting wires and SRR. This indicates that the negative $\varepsilon_{\text{eff}}(\omega)$ of the straight wires is combined with the negative $\mu_{\text{eff}}(\omega)$ of SRR to allow the propagation, and present the novel composite material with negative permittivity/permeability.

This performance can be compared with the behavior of the composite material of straight wires/SRR for the $H_\perp$ case (Fig. 8.13), as determined in Fig. 8.14. As observed, in this case, a small pass-band is obtained which is within a narrow range that ends...
abruptly at the band edge of the lowest dielectric gap of the SRR. The pass-band occurs where the effective dielectric of the SRR exceeds the negative dielectric of the medium of wires (positive permittivity/permeability). The contrast between the propagation bands of $H_\parallel$ and $H_\perp$ cases, demonstrates the difference between the performances of SRR ($H_\parallel$) with negative permeability compared to the SRR ($H_\perp$) with negative permittivity. Note that the observations are very similar compared to the data presented in [13] utilizing dispersion diagram.

Fig. 8.3: Electromagnetic performances of the media having (a) negative effective permittivity, (b) negative effective permeability, and (c) both negative effective permittivity/permeability.
Fig. 8.4: 2-Layer conducting straight wires: (a) Periodic structure, (b) Unit cell of the structure. Note that the PBC walls are positioned on the PEC edges.

Fig. 8.5: Normal and 30° oblique incidence reflected power of the straight wires. Negative effective permittivity of the structure creates the dielectric gap region.
Fig. 8.6: 2-Layer split ring resonators ($H_i$): (a) Periodic structure, (b) Unit cell of the structure.

Fig. 8.7: Normal and 30° oblique incidence reflected power of the SRR ($H_i$). Negative effective permeability of the structure creates the magnetic gap regions.
Fig. 8.8: 2-Layer split ring resonators ($H_\perp$-same dimensions as $H_\parallel$ case): (a) Periodic structure, (b) Unit cell of the structure.

Fig. 8.9: Normal and 30° oblique incidence reflected power of the SRR ($H_\perp$). Negative effective permittivity of the structure creates the dielectric gap regions.
Fig. 8.10: Composite material of 2-layer straight wires/split ring resonators ($H_{||}$): (a) Periodic structure, (b) Unit cell of the structure. Note that the PBC walls are positioned on the PEC edges.

Fig. 8.11: Normal and $30^\circ$ oblique incidence reflected power of the straight wires/SRR ($H_{||}$). Negative effective permittivity of the straight wires are combined with the negative effective permeability of the SRR ($H_{||}$) to present the LH material around the frequency $5.3\, GHz$ (pass-band region).
Fig. 8.12: Illustration of the LH material with negative permittivity/permmeability. The pass-band occurs within the dielectric/magnetic gap regions of the straight wires/SRR.
Fig. 8.13: Composite material of 2-layer straight wires/split ring resonators ($H_\perp$): (a) Periodic structure, (b) Unit cell of the structure. Note that the PBC walls are positioned on the PEC edges.

Fig. 8.14: Normal and $30^\circ$ oblique incidence reflected power of the straight wires/SRR ($H_\perp$). The SRR ($H_\perp$) has the positive permeability and no LH property can be achieved. Notice that the pass-band occurs where the composite material has the positive effective permittivity/permeability.
8.3 Summary and Future Investigations

In this chapter, a novel composite material of periodic conducting straight wires/split ring resonators is presented to introduce a LH material with simultaneously negative permittivity and permeability. The powerful FDTD/Prony technique is successfully applied to characterize and demonstrate the unique electromagnetics phenomena of the complex structure. It is illustrated that the straight wires with negative permittivity are properly combined with the split rings with negative permeability to design a composite material with negative permittivity/permeability.

The results presented in this work, are the basis for further investigations of the fascinating electrodynamics effects anticipated for such composite material in order to incorporate these into some potential applications.
Chapter 9

Summary and Suggestions for Future Work

9.1 Summary

The main objectives of the work described in this dissertation are to characterize the electromagnetic performances of two classes of composite structures, namely, (a) multi-layered complex media, and (b) periodic Electromagnetic Band-Gap (EBG) structures; and to incorporate their unique propagation characteristics into some novel applications. The advanced and diversified computational techniques are developed to effectively analyze the interaction of the structures with electromagnetic waves. The key achievements of this research may be summarized as

**Vector Wave Solution/GA Technique:** The vector wave solution of Maxwell’s equations is integrated with the genetic algorithm to obtain a capable technique for characterizing multi-layered complex media and presenting their novel optimized designs.

**RCS Reduction of Canonical Targets:** The modal solution/GA technique is applied to determine the optimal composite coatings for RCS reduction of canonical targets in a wide-band of frequency range. Both monostatic and bistatic cases are studied. A Hybrid
GA with planar/curved surface implementation is also introduced to efficiently reduce the RCS of the curved structures.

**Non-Uniform Luneburg and 2-Shell Lens Antennas:** Design optimization of the non-uniform Luneburg lens antenna is successfully obtained utilizing the vector wave solution of Maxwell’s equations integrated with the GA/adaptive-cost-function. Additionally, a novel 2-shell lens antenna with high performance radiation characteristics is presented. Many useful engineering guidelines are highlighted.

**FDTD Analysis of Complex Periodic Structures:** A powerful computational engine utilizing FDTD technique with PBC/PML boundary conditions is developed to accurately determine the broadband electromagnetic characteristics of the complex periodic EBG structures. The Prony extrapolation scheme is also integrated to increase the efficiency of the presented method.

**Frequency Selective Surfaces:** The FDTD/Prony technique is successfully applied to characterize different classes of challenging FSS such as, (a) double concentric square loop FSS, (b) dipole FSS manifesting high $Q$ resonance, (c) patch FSS manifesting high $Q$ resonance, (d) crossed dipole FSS, (e) dichroic plate FSS, (f) inset self-similar aperture FSS, (g) fractal FSS, (h) multi-layered tripod FSS, (i) 9-layer dielectric/inductive square loop FSS, and (j) smart surface mushroom FSS. The results are extremely satisfactory demonstrating the capability of the developed engine.
**Photonic Band-Gap Materials:** To understand the performance of the PBG/periodic dielectric materials, the FDTD/Prony technique is applied to present their innovative propagation characteristics. The band-gap phenomena of the triangular, rectangular, and woodpile PBG are effectively investigated. The results are integrated into the three potential applications, namely, high $Q$ nanocavity lasers, guiding the EM waves in sharp bends, and miniaturized microstrip patch antennas. A comparative study with the effective dielectric materials is also provided.

**Composite LH Media:** A composite material of two periodic conducting structures, straight wires with negative effective permittivity/split ring resonators with negative effective permeability, is presented to generate a new medium with both negative permittivity/permeability. The structure has the unique electrodynamic properties, and is successfully characterized utilizing the FDTD/Prony technique.

### 9.2 Suggestions for Future Work

It is demonstrated that the computational engines presented in this dissertation are very capable in characterizing complex layered and periodic band-gap structures. However, it is believed that the number of new ideas is endless. The followings are a few suggestions for future work:

- In this work, to efficiently reduce the RCS of curved structures, a GA with hybrid planar/curved surface implementation is introduced. The hybrid GA is successfully applied to determine the optimal coating for a canonical spherical
structure. Although, the obtained lossy RAM coating can be also applied to reduce the RCS of curved/non-canonical structures; however, one may extend the developed method to directly integrate the hybrid GA with a numerical technique analyzing surfaces with arbitrary curvatures, to effectively present their optimal coatings.

• To present the optimal design for some applications of interest, one needs to determine the tradeoffs between a set of conflict goals. The GA with adaptive cost function, as applied in this work, has the potential to properly obtain the best tradeoffs. However, finding a suitable algebraic combination of goals for use as an adaptive objective function may be a difficult task. The Pareto GA’s avoid this problem by returning not a single optimal design, but the set of designs that represents all of the best tradeoffs [20]. This can be achieved by modifying the genetic algorithm, described in this work, to allow the population to converge toward the whole Pareto front instead of a subset of that.

• The FDTD/Prony technique, as demonstrated in this dissertation, is a highly successful solution to a wide variety of electromagnetic wave interaction problems. However, the developed method is restricted to the orthogonal grids, and the structure boundaries that do not conform to the orthogonal coordinate must be approximated using staircasing. The Nonorthogonal FDTD (N-FDTD) scheme based on the global curvilinear coordinate system [9] is an effective approach to accurately analyze these structures. In addition, to improve the
efficiency of the technique, one may enmesh the FDTD domain using the nonuniform grids [9]. Thus, the next challenge for this work could be the implementation of a very capable computational engine utilizing N-FDTD/nonuniform grids integrated with the Prony method to accurately and efficiently obtain the EM performance of the structures with complex boundaries.

- The characteristics of different classes of challenging periodic electromagnetic band-gap structures are presented in this research. However, the key engineering question is, how one can present the best optimal design for the application of interest? To answer this question, our FDTD/Prony engine has to be integrated with a capable optimization method. The optimizer could be the genetic algorithm, or a local technique such as conjugate-gradient or quasi-Newton method. The main considerations are (a) to properly choose the objective function, and (b) to efficiently integrate the FDTD with the optimization technique. The FDTD/optimization method has a tremendous advantage in presenting new structural configurations for frequency selective surfaces, photonic band-gap materials, etc.

- The performance of the novel LH composite material, presented in this work, demonstrates that the frequency bandwidth of the region with negative permittivity/permeability and the level of reflection coefficient in this region, are the important issues that need to be studied in greater depth. Further work is required to investigate a more applicable structure with improved characteristics
and simple fabrication. The unique electrodynamic properties of the LH media can be integrated into the future potential designs.
Appendix A

Electromagnetic Scattering from an Arbitrary Configuration of Eccentric Spheres

A.1 Objective

In this appendix the formulation for the scattering of the electromagnetic waves from an arbitrary configuration of eccentric spheres is presented. The modal solution of Maxwell’s equations in the coordinate of each sphere is represented using the spherical vector wave functions. Next, the Green’s second theorem is used to impose the boundary conditions on the spherical interfaces. Additional theorems in the spherical coordinates are also applied to express the $M$ and $N$ wave functions in one coordinate system in terms of the wave functions in another coordinate system. Utilizing this transformation and by employing the orthogonality of the spherical wave functions, one can obtain the matrix equations, and then the scattered fields.
A.2 Modal Solution of Eccentric Spherical Structures

Fig. A.1 depicts a configuration of $N$ eccentric spheres illuminated by an arbitrary incident plane wave. The EM fields in the $i^{th}$ region and spherical coordinate $o_i$ can be expressed utilizing the spherical vector wave functions (Chapter 2) in the following form

$$E_i(r) = \sum_{n,m} C_{i,nm} \cdot W_{n,m}^T(k_i, r_j)$$  \hspace{1cm} (A.1)

where

$$C_{i,nm} = (a_{i,nm} \quad b_{i,nm} \quad c_{i,nm} \quad d_{i,nm})$$  \hspace{1cm} (A.2a)

$$W_{n,m}(k_i, r_j) = (M_{n,m}^{(1)}(k_i, r_j) \quad M_{n,m}^{(4)}(k_i, r_j) \quad N_{n,m}^{(1)}(k_i, r_j) \quad N_{n,m}^{(4)}(k_i, r_j)).$$  \hspace{1cm} (A.2b)
In region 1, the EM fields are the combination of both incident and scattered fields. For the incident field propagating in the \( \hat{i} \) direction with the polarization along the \( \hat{e} \), a representation in the spherical coordinate \( o_i \) can be obtained as

\[
E_i^i(r) = \sum_{n,m} C_{1,nm}^i \cdot W_{n,m}(k_1, r_1).
\]  

(A.3)

The unknown coefficient \( C_{1,mm}^i \) is

\[
C_{1,mm}^i = \begin{pmatrix} a_{1,mm}^i & 0 & c_{1,mm}^i & 0 \end{pmatrix}
\]

(A.4)

where [15]

\[
a_{1,mm}^i = \frac{j^{-n} (-1)^n (2n + 1)}{\sqrt{n(n+1)}} \hat{e} \cdot R_{n, -m} (\theta', \phi')
\]

(A.5a)

\[
c_{1,mm}^i = \frac{j^{-n-1} (-1)^{m+1} (2n + 1)}{\sqrt{n(n+1)}} \hat{e} \cdot S_{n, -m} (\theta', \phi')
\]

(A.5b)

and

\[
R_{n, m}(\theta, \phi) = \frac{1}{\sqrt{n(n+1)}} e^{im\phi} \left[ \hat{\theta} \cdot \frac{j m}{\sin \theta} P_n^m (\cos \theta) - \hat{\phi} \frac{d P_n^m (\cos \theta)}{d \theta} \right]
\]

(A.6a)

\[
S_{n, m}(\theta, \phi) = \frac{1}{\sqrt{n(n+1)}} e^{im\phi} \left[ \hat{\theta} \cdot \frac{d P_n^m (\cos \theta)}{d \theta} + \hat{\phi} \frac{j m}{\sin \theta} P_n^m (\cos \theta) \right].
\]

(A.6b)

Thus, the total field in region 1 and coordinate \( o_i \) is simply determined using the Eq. (A.1) with coefficient

\[
C_{1,mm} = \begin{pmatrix} a_{1,mm} & b_{1,mm} & c_{1,mm} & d_{1,mm} \end{pmatrix}.
\]

(A.7)

The scattered fields in regions \( i = 2, 3, \ldots, N \) is obtained using Eq. (A.1) with

\[
C_{i,mm} = \begin{pmatrix} a_{i,mm} & 0 & c_{i,mm} & 0 \end{pmatrix}.
\]

(A.8)
A.3 Boundary Conditions

A.3.1 Dyadic Green’s Second Theorem

Boundary conditions on the spherical interfaces are imposed using the dyadic Green’s second theorem in the region between the spheres with propagation constant $k$

$$\int_s \left[ P \cdot \nabla' \times \nabla' \times Q - (\nabla' \times \nabla' \times P) \cdot Q \right] dv' = -\int_s \left[ (\hat{n}' \times \nabla' \times P) \cdot Q + (\hat{n}' \times P) \cdot \nabla' \times Q \right] ds'. \quad (A.9)$$

Taking

$$P = H \quad (A.10a)$$

$$Q = G(r', r) = \left(1 + \frac{1}{k^2} \nabla' \nabla' \cdot \right) G(r, r') \hat{1} , \quad G(r, r') = \frac{e^{-jk|r-r'|}}{|r-r'|} \quad (A.10b)$$

one can obtain for $r \not\in V$

$$L(r) \equiv \hat{n} \times \nabla \times \int_s (\hat{n}' \times H) \cdot G ds' + j \omega \epsilon \hat{n} \times \int_s (\hat{n}' \times E) \cdot G ds' = 0. \quad (A.11)$$

The dyadic Green’s function $G(r', r)$ can be expressed in terms of spherical wave functions $M$ and $N$ as

$$G(r', r) = -jk \sum_{n,m} (-1)^m \frac{2n+1}{n(n+1)} \left( M_{n-m}^{(1)} (r_<) M_{n,m}^{(4)} (r_>) + N_{n-m}^{(1)} (r_<) N_{n,m}^{(4)} (r_>) \right) \quad (A.12)$$

where $>$ or $<$ means that the spherical variables $(r, \theta, \phi)$ are defined in the coordinate $r$ or $r'$ based on the $Max(r, r')$ or $Min(r, r')$, respectively. By substituting Eq. (A.12) in Eq. (A.11), the following equations for $r \in S_i \ (i = 1, 2, \cdots, N)$ are attained [86]
\[ r > r' : \]
\[
\hat{n} \times \left[ \sum_{n,m} (-1)^m \frac{2n+1}{\sqrt{n(n+1)}} \left( N^{(4)}_{n,m}(k,r) j_n(kr) \int_S (\hat{n}' \times H) \cdot R_{n,-m} \, d\Omega' + M^{(4)}_{n,m}(k,r) j_n(kr) \int_S (\hat{n}' \times E) \cdot R_{n,-m} \, d\Omega' \right) \right] = 0 \quad (A.13a)
\]
\[ r < r' : \]
\[
\hat{n} \times \left[ \sum_{n,m} (-1)^m \frac{2n+1}{\sqrt{n(n+1)}} \left( N^{(1)}_{n,m}(k,r) h^{(2)}_n(kr) \int_S (\hat{n}' \times H) \cdot S_{n,-m} \, d\Omega' + M^{(1)}_{n,m}(k,r) h^{(2)}_n(kr) \int_S (\hat{n}' \times E) \cdot S_{n,-m} \, d\Omega' \right) \right] = 0 \quad (A.13b)
\]
where \( d\Omega' \) is the solid angle of sphere. Notice that \( z_n'(x) \equiv (xz_n(x))' \), where \( z_n \) could be spherical Bessel or second order spherical Hankel functions; and
\[
\hat{r} \times M^{(1)}_{n,m}(k,r) = \sqrt{n(n+1)} j_n(kr) S_{n,m}(\theta, \phi) \quad (A.14a)
\]
\[
\hat{r} \times M^{(4)}_{n,m}(k,r) = \sqrt{n(n+1)} h^{(2)}_n(kr) S_{n,m}(\theta, \phi) \quad (A.14b)
\]
\[
\hat{r} \times N^{(1)}_{n,m}(k,r) = -\sqrt{n(n+1)} \frac{j_n(kr)}{kr} R_{n,m}(\theta, \phi) \quad (A.14c)
\]
\begin{equation}
\hat{r} \times \mathbf{N}_{n,m}^{(4)}(k, \mathbf{r}) = -\sqrt{n(n+1)} \frac{\hat{h}_n^{(2)}(kr)}{kr} \mathbf{R}_{n,m}(\theta, \phi).
\tag{A.14d}
\end{equation}

Using Eq. (A.13), continuity of \( \hat{n}' \times \mathbf{E} \) and \( \hat{n}' \times \mathbf{H} \) on the spherical boundaries, and the orthogonality between \( \mathbf{R}_{n,m} \) and \( \mathbf{S}_{n,m} \), it is obtained that

\[ r \in S_j, \; j = 1, 2, \cdots, N : \]

\begin{align*}
&\sum_{n,m} r_i^2 b_{i,nn} \left( -\frac{j}{\eta_i} h_n^{(2)}(kr_i) \frac{h_n^{(2)}(kr_i)}{k_r} + \frac{j}{\eta} \frac{h_n^{(2)}(kr_i)}{k_r} h_n^{(2)}(kr_i) \right) \hat{r} \times \mathbf{N}_{n,m}^{(1)}(k, r_i) + \\
&\sum_{n,m} r_i^2 a_{i,nn} \left( \frac{j}{\eta_i} \frac{j_n(kr_i)}{k_r} j_n(kr_i) - \frac{j}{\eta} \frac{j_n(kr_i)}{k_r} j_n(kr_i) \right) \hat{r} \times \mathbf{N}_{n,m}^{(4)}(k, r_i) + \\
&\sum_{n,m} r_i^2 a_{i,nn} \left( \frac{j}{\eta_i} \frac{j_n(kr_i)}{k_r} j_n(kr_i) + \frac{j}{\eta} \frac{j_n(kr_i)}{k_r} j_n(kr_i) \right) \hat{r} \times \mathbf{N}_{n,m}^{(4)}(k, r_i) = \\
&\sum_{n,m} r_i^2 a_{i,nn}' \left( -\frac{j}{\eta_i} h_n^{(2)}(kr_i) \frac{j_n(kr_i)}{k_r} + \frac{j}{\eta} \frac{h_n^{(2)}(kr_i)}{k_r} j_n(kr_i) \right) \hat{r} \times \mathbf{N}_{n,m}^{(1)}(k, r_i) + \\
&\sum_{n,m} r_i^2 a_{i,nn}' \left( \frac{j}{\eta_i} \frac{j_n(kr_i)}{k_r} j_n(kr_i) - \frac{j}{\eta} \frac{j_n(kr_i)}{k_r} j_n(kr_i) \right) \hat{r} \times \mathbf{N}_{n,m}^{(4)}(k, r_i) + \\
&\sum_{n,m} r_i^2 a_{i,nn}' \left( \frac{j}{\eta_i} \frac{j_n(kr_i)}{k_r} j_n(kr_i) + \frac{j}{\eta} \frac{j_n(kr_i)}{k_r} j_n(kr_i) \right) \hat{r} \times \mathbf{N}_{n,m}^{(4)}(k, r_i) + \\
&\sum_{n,m} r_i^2 a_{i,nn}' \left( \frac{j}{\eta_i} h_n^{(2)}(kr_i) \frac{j_n(kr_i)}{k_r} - \frac{j}{\eta} \frac{h_n^{(2)}(kr_i)}{k_r} j_n(kr_i) \right) \hat{r} \times \mathbf{N}_{n,m}^{(1)}(k, r_i). \tag{A.15}
\end{align*}
A.3.2 Spherical Translational Additional Theorems

To apply the orthogonality of the spherical vector wave functions and obtain the matrix equations, one needs to express the spherical functions \( M \) and \( N \) in one coordinate system in terms of the spherical functions in another coordinate system, based on the translational additional theorems in spherical coordinates [87], [88]. After some mathematical manipulations, it is determined that

\[
M_{n,m}(k, r_j) = \sum_{\nu, \mu} \left( A_{\nu \mu i j}^{nm} M_{\nu \mu}(k, r_j) + B_{\nu \mu i j}^{nm} N_{\nu \mu}(k, r_j) \right)
\]  
(A.16a)

\[
N_{n,m}(k, r_j) = \sum_{\nu, \mu} \left( A_{\nu \mu i j}^{nm} N_{\nu \mu}(k, r_j) + B_{\nu \mu i j}^{nm} M_{\nu \mu}(k, r_j) \right)
\]  
(A.16b)

where

\[
A_{\nu \mu i j}^{nm} = (-1)^{\mu} \sum_p a(n, m, \nu, -\mu, p)a(n, \nu, p)z_p(kr_0^{ij})P_{p}^{m-\mu}(\cos \theta_0^{ij})e^{i(m-\mu)\phi_0^{ij}}
\]  
(A.17a)

\[
B_{\nu \mu i j}^{nm} = (-1)^{\mu} \sum_p a(n, m, \nu, -\mu, p, p-1)b(n, \nu, p)z_p(kr_0^{ij})P_{p}^{m-\mu}(\cos \theta_0^{ij})e^{i(m-\mu)\phi_0^{ij}}
\]  
(A.17b)

\[
a(n, \nu, p) = j^{n+\nu-p} \left[ 2\nu(\nu+1)(2\nu+1) + (\nu+1)(n-\nu+p+1)(n+\nu-p) - \nu(n-p+1)(n+\nu+p+2) \right]/(2\nu(\nu+1))
\]  
(A.17c)

\[
b(n, \nu, p) = j^{n+\nu-p} \left[ (n+\nu+p+1)(n+\nu-p+1)(\nu-n+p)(n-\nu+p) \right]^{1/2} \cdot (2\nu+1)/(2\nu(\nu+1))
\]  
(A.17d)

\[
a(n, m, \nu, \mu, p) = (-1)^{m+\mu} (2p+1) \left[ \frac{(n+m)!(\nu+\mu)!(p-m-\mu)!}{(n-m)!(\nu-\mu)!(p+m+\mu)!} \right]^{1/2} \cdot \left( \begin{array}{ccc} n & \nu & p \\ 0 & 0 & 0 \\ m & \mu & -m-\mu \end{array} \right)
\]  
(A.17e)
\[ a(n, m, \nu, \mu, p, q) = (-1)^{m+\mu} (2p + 1) \left[ \frac{(n+m)!(\nu+\mu)!(p-m-\mu)!}{(n-m)!(\nu-\mu)!(p+m+\mu)!} \right]^{1/2}. \]

\[
\begin{pmatrix}
\frac{n}{p} & \frac{\nu}{ \mu } & \frac{q}{ -m-\mu } \\
0 & 0 & 0 \\
\end{pmatrix}
\]

(A.17f)

\[
\begin{pmatrix}
j_1 & j_2 & j_3 \\
l_1 & l_2 & l_3 \\
\end{pmatrix}
\]

is the Wigner 3-j symbol, and \((r_0^{ij}, \theta_0^{ij}, \phi_0^{ij})\) is the spherical coordinate of the position vector \(r_j\) with respect to the coordinate \(r_i\).

### A.3.3 Matrix Equations

Substituting Eq. (A.16) into the Eq. (A.15), and using the orthogonal properties of the spherical function, the following matrix equations are obtained

\[
r \in S_j, j = 1, 2, \cdots, N:
\]

1) \[
j_v^d(kr_j) \sum_{n,m} A_{np}^{nm,ij} \frac{r_i}{r_j} \left( \frac{1}{k_i \eta_i} h_n^{(2)d}(kr_i) h_n^{(2)d}(k_r_i) - \frac{1}{k_\eta} h_n^{(2)d}(kr_i) h_n^{(2)d}(k_r_i) \right) h_{1,nn} +
\]

\[
B_{np}^{nm,ij} \frac{r_i}{r_j} \left( \frac{1}{k_i \eta_i} h_n^{(2)d}(kr_i) h_n^{(2)d}(k_r_i) + \frac{1}{k_\eta} h_n^{(2)d}(kr_i) h_n^{(2)d}(k_r_i) \right) d_{i,nn} +
\]

\[
h_v^{(2)d}(kr_j) \sum_{i=2}^{N} A_{ij}^{nm,ij} \frac{r_i}{r_j} \left( -\frac{1}{k_i \eta_i} j_n(kr_i) j_n^{(d)}(k_r_i) + \frac{1}{k_\eta} j_n(kr_i) j_n^{(d)}(k_r_i) \right) a_{i,nn} +
\]

\[
B_{ij}^{nm,ij} \frac{r_i}{r_j} \left( -\frac{1}{k_i \eta_i} j_n(kr_i) j_n^{(d)}(k_r_i) - \frac{1}{k_\eta} j_n(kr_i) j_n^{(d)}(k_r_i) \right) c_{i,nn} =
\]

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\[-j^d (kr_j) \sum_{n,m} A_{n,m,j}^{nm} \sum_{n,m} r_i n \frac{r_j}{r_i} \left( \frac{1}{k_i \eta_i} h_n^{(2)} (kr_i) j_n^d (k_i r_i) - \frac{1}{k \eta} h_n^{(2)} (k r_i r_i) j_n^d (k_i r_i) \right) a_{i,nn}^i + \]

\[B_{n,m,j}^{nm} \sum_{n,m} r_i n \frac{r_j}{r_i} \left( - \frac{1}{k_i \eta_i} h_n^{(2)} (kr_i) j_n^d (k_i r_i) + \frac{1}{k \eta} h_n^{(2)} (k r_i r_i) j_n^d (k_i r_i) \right) c_{i,nn}^i \]  

(A.18a)

II)

\[j^d (kr_j) \sum_{n,m} B_{v}^{nm,n} \sum_{n,m} \sum_{i=2}^{N} r_i n \frac{r_j}{r_i} \left( \frac{1}{k_i \eta_i} j_n (kr_i) j_n^d (k_i r_i) - \frac{1}{k \eta} j_n^d (k r_i r_i) j_n (k_i r_i) \right) a_{i,nn} + \]

\[A_{v}^{nm,n} \sum_{n,m} \sum_{i=2}^{N} r_i n \frac{r_j}{r_i} \left( - \frac{1}{k_i \eta_i} j_n^d (k r_i r_i) j_n (k_i r_i) + \frac{1}{k \eta} j_n (k r_i r_i) j_n^d (k_i r_i) \right) c_{i,nn} = \]

\[-j^d (kr_j) \sum_{n,m} B_{v}^{nm,n} \sum_{n,m} \sum_{i=2}^{N} r_i n \frac{r_j}{r_i} \left( \frac{1}{k_i \eta_i} h_n^{(2)} (kr_i) j_n^d (k_i r_i) + \frac{1}{k \eta} h_n^{(2)} (k r_i r_i) j_n^d (k_i r_i) \right) a_{i,nn} + \]

\[A_{v}^{nm,n} \sum_{n,m} \sum_{i=2}^{N} r_i n \frac{r_j}{r_i} \left( - \frac{1}{k_i \eta_i} h_n^{(2)} (k r_i r_i) j_n (k_i r_i) + \frac{1}{k \eta} h_n^{(2)} (k r_i r_i) j_n^d (k_i r_i) \right) c_{i,nn}^i . \]  

(A.18b)

This set of matrix equations can be successfully solved to determine the unknown coefficients \(b_{i,nn}, d_{i,nn}, a_{i,nn}, \) and \(c_{i,nn} \) (\(i = 2, 3, \cdots, N\)); and then compute the scattered electromagnetic fields from the eccentric spheres.
Appendix B

FDTD/Prony for Efficient Analysis of Complex Periodic Structures

As mentioned earlier, the periodic structures require extensive simulation time due to the multiple reflections, which occur in setting up the modal distribution on the periodic body. Thus, the simulation involves many time steps, which is time consuming and it increases the numerical errors over the long period of time. The periodic structures can be efficiently analyzed utilizing the Prony’s scheme.

To this end, the FDTD technique is run long enough to allow a sufficient time for settling the initial transient and developing the time harmonic fields. Next, the Prony’s method is applied to effectively extrapolate the truncated time domain results. Based on our experience, to achieve the accurate results, one needs to allow the process of the time-stepping algorithm for about ten-fifteen round trips propagation of the scattered waves between the ABC walls.

Fig. B.1 shows a double concentric square loop FSS. The structure is analyzed utilizing the FDTD and FDTD/Prony techniques; and the results for the scattered fields in time and frequency domains are determined in Figs. B.2 and B.3. To accurately characterize the structure using the FDTD technique, one needs about 28 cycles (round
trips) time analysis between the ABC walls, which it takes around 155 min. on an HPC-180 workstation.

To properly apply the Prony’s method, the FDTD technique is run for about ten cycles (55 min.) including of 2124 iterations, as shown in Fig. B.2. A window of 600 iterations is sampled at every 12 iterations to obtain 50 data points from which the complex exponential functions and unknown coefficients are found by applying the Prony’s method. After experimenting a few times by shifting the window up and down on the FDTD records, it is obtained that a 5-terms series of exponential functions satisfactorily fits the FDTD data with mean square error $\chi = 10^{-6}$ [89]. The scattered field, as demonstrated in Figs. B.2 and B.3, is accurately and efficiently obtained utilizing the FDTD/Prony in about 55.04 min. Notice that the FDTD with ten cycles propagating waves between the ABC walls cannot generate the accurate results, as illustrated in Fig. B.3.

Fig. B.1: Double concentric square loop FSS: (a) Periodic structure, (b) Unit cell of the structure.
Fig. B.2: Normal incidence scattered field of the double concentric square loop FSS in the time domain. A comparative study between FDTD and FDTD/Prony.

Fig. B.3: Normal incidence reflected power of the double concentric square loop FSS in the frequency domain. A comparative study between FDTD and FDTD/Prony.
Bibliography


